SECTION I, MECHANICS

1. **E**

The initial velocity is negative, so the initial displacement must start out negative. The object also continues at a constant speed after time t_1, so that position vs. time should be a straight line with a positive slope at this time. Choice (E) is the only answer choice that meets these two conditions.

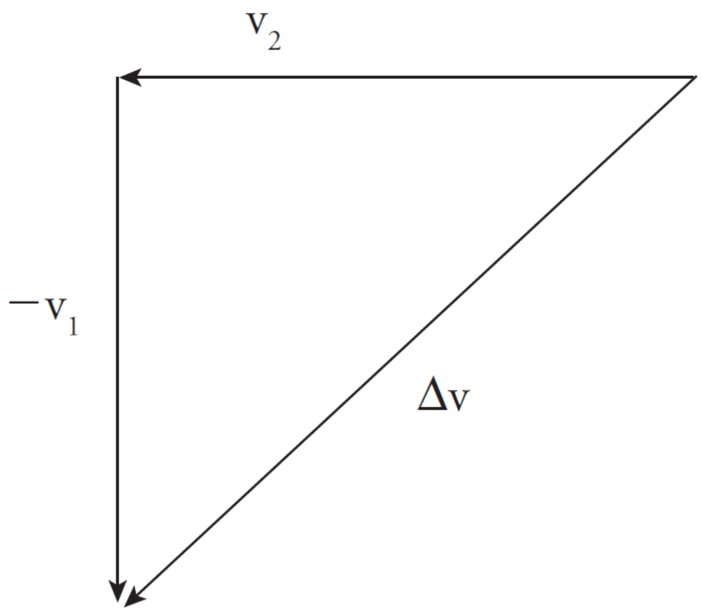
2. **B**

When an object is dropped its initial vertical velocity is zero. We can solve for the time it takes to land using the following constant acceleration equation and plugging in appropriate values.

$$y = y_o + v_0 t + \frac{1}{2} g t^2$$
$$0 = 80 + 0 + \frac{1}{2}(-10)t^2$$
$$t = 4\,\text{s}$$

3. **D**

The direction of the change in momentum of an object will be the direction of $\Delta\mathbf{v}$. The resultant of $\mathbf{v}_2 - \mathbf{v}_1$ is shown below.



4. **B**

The acceleration of the $2m$ mass is the same as the acceleration of the m and $3m$ mass because they are connected to each other. Use Newton's Second Law on the entire system to solve for the acceleration.

$$\sum F = ma$$
$$F = (3m + 2m + m)a$$
$$a = \frac{F}{6m}$$

5. **B**

Read the force directly off the graph. At t = 2 s the force is 3 N. Use Newton's Second Law to solve for the acceleration.

$$\sum F = m a$$
$$3 = 2a$$
$$a = 1.5 \text{ m/s}^2$$

6. **C**

We will use the impulse–momentum theorem here. We cannot use the constant acceleration equations with the acceleration from question 5 because the acceleration is not constant. The area between the graph and the x-axis is the impulse.

$$\int F\,dt = \Delta p$$
$$\text{Area} = m\Delta v$$
$$\frac{1}{2}(3)(9) = 2(v_2 - v_1)$$
$$13.5 = 2v_2$$
$$v_2 = 6.75 \text{ m/s}$$

7. **D**

We will use the Law of Conservation of Energy for this problem. The cylinder has rotational and translational kinetic energy as it rolls along the horizontal surface. All of that energy will be converted into gravitational potential energy when it reaches its maximum height up the ramp.

$$K_R + K_T = U_g$$
$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgh$$
$$\frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$
$$\frac{1}{4}mv^2 + \frac{1}{2}mv^2 = mgh$$
$$\frac{3}{4}v^2 = gh$$
$$h = \frac{3v^2}{4g}$$

8. **A**

Again we will use the Law of Conservation of Energy. The easiest way to determine which object will reach a greater height is to determine which object has more kinetic energy as it rolls along the horizontal surface. If both the cylinder and the hoop have the same velocity then the hoop will have more total kinetic energy because it has a greater rotational inertia. The calculation at the top of the next page shows the calculation for the height which also shows that the hoop reaches a greater height.

$$K_R + K_T = U_g$$
$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgh$$
$$\frac{1}{2}\left(mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$
$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mgh$$
$$v^2 = gh$$
$$h = \frac{v^2}{g}$$

While both (A) and (D) indicate that the hoop reaches a greater maximum height than the cylinder, (D) is incorrect as the cylinder reaches 3/4 the height of the hoop.

9. **D**

Find the periods of each pendulum.

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}} = 2\pi\sqrt{\frac{0.9\text{ m}}{10\text{ m/s}^2}} = 0.6\pi\text{ s}$$

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}} = 2\pi\sqrt{\frac{1.6\text{ m}}{10\text{ m/s}^2}} = 0.8\pi\text{ s}$$

The hanging masses will make contact when both pendulums have completed a cycle at the same moment. This will occur at 2.4π s, at which point the first pendulum will have completed 4 cycles.

10. **D**

For all three of these problems, (A) cannot be the correct choice because the gravitational force mg must be directed straight down. Choice (B) cannot be the correct choice because the normal force N must be perpendicular to the surface. If the box is going up the incline then the frictional force must be directed down the incline, which is only true for (D).

11. **C**

For all three of these problems, (A) cannot be the correct choice because the gravitational force mg must be directed straight down. Choice (B) cannot be the correct choice because the normal force N must be perpendicular to the surface. If the box is moving at a constant velocity the net force in the x-direction, along the incline, must be zero. This is true only for (C).

12. **E**

For all three of these problems, (A) cannot be the correct choice because the gravitational force mg must be directed straight down. Choice (B) cannot be the correct choice because the normal force N must be perpendicular to the surface. If the box is speeding up while moving down the incline then the net force must be directed down the incline, and the frictional force must be directed up the incline to oppose the motion. This is true only for (E).

13. **E**

According to the impulse–momentum theorem, the change in momentum of an object is equal to the impulse on an object. The impulse on an object is equal to the integral of $F \cdot dt$.

$$\int F \; dt = \text{change in momentum}$$

$$\int_0^{t_1} Ct^3 \; dt = \text{change in momentum}$$

$$\left.\frac{Ct^4}{4}\right|_0^{t_1} = \text{change in momentum}$$

$$\frac{Ct_1^4}{4} = \text{change in momentum}$$

14. **B**

To reduce this elliptical orbit to a circular orbit, the spaceship must slow down. To accomplish this, the engine burst must be in the same direction of the velocity which will cause the spaceship to accelerate in the opposite direction as per Newton's Third Law of Motion (action/reaction). This will cause the spaceship to slow down. The velocity at point P is directed to the left, so the engine burst must be to the left.

15. **A**

The normal force of the track would point toward the center of the circle for the entire time the motorcycle was in the circular loop and the velocity would always be tangent to the path. The normal force and the displacement of the motorcycle are always perpendicular to each other. Therefore, the work done by the normal force would be zero since work is the dot product of the force and the displacement.

16. **D**

The speed of the center of mass of the ball can be calculated using the equation, $x = vt$.Inserting the appropriate numbers gives

$$30\text{m} = v(10\text{s})$$
$$v = 3 \text{ m/s}$$

The angular speed of the ball about the contact point can be calculated using the equation, $v = \omega r$. The value for r is the distance from the contact point to the center of mass, which is the radius of the ball. Inserting the appropriate numbers gives

$$3 \text{ m/s} = \omega\,(0.2 \text{ m})$$

$$\omega = 15 \text{ rad/s}$$

17. **B**

When objects stick together, a perfectly inelastic collision has occurred. For this situation, kinetic energy is not conserved. This eliminates (C) and (D). Linear momentum is conserved when the net force on the system is zero. In this situation the hinge in the bar is exerting a force on it prohibiting it from translating to the right. Therefore linear momentum is not conserved. This eliminates (A) and (E). Angular momentum is conserved when the net torque on a system is zero. The force exerted by the hinge provides no torque because the lever arm is zero. Therefore, (B) is correct.

18. **B**

The period for a spring-mass system is given by the equation $T = 2\pi\sqrt{\frac{m}{k}}$. Use that equation to solve for the mass, as shown below.

$$4s = 2\pi\sqrt{\frac{m}{8\text{N/m}}}$$
$$\frac{2}{\pi} = \sqrt{\frac{m}{8}}$$
$$\frac{4}{\pi^2} = \frac{m}{8}$$
$$\frac{32}{\pi^2} = m$$

19. **C**

The equation for an object undergoing simple harmonic motion will be of the form $x = A\cos(\omega t + \phi)$, where A is the amplitude, ω is the angular frequency and ϕ is the phase constant. None of the solutions involve the phase constant, so we will ignore that part of the equation. The amplitude of the oscillation is 5 cm, which is 0.05 m. This eliminates (D). The angular frequency can be calculated by using the fact that the period is 2π divided by the angular frequency.

$$T = \frac{2\pi}{\omega}$$
$$4 = \frac{2\pi}{\omega}$$
$$\omega = \frac{\pi}{2}$$

20. **B**

When the object is 10 m above the structure, it has $PE = mgh = 200$ J relative to the top of the structure. Crashing through the first barrier uses 50 J of energy, leaving 150 J. Then the object gains another 20 J as it falls, meaning it has 170 J when it strikes the second barrier. This pattern will continue as shown below:

$$200 \rightarrow 150 \rightarrow 170 \rightarrow 120 \rightarrow 140 \rightarrow 90 \rightarrow 110 \rightarrow 60 \rightarrow 80 \rightarrow 30 \rightarrow 50 \rightarrow 0$$

During this process, the object experiences 6 of the 50 J reductions, so it breaks through 6 barriers.

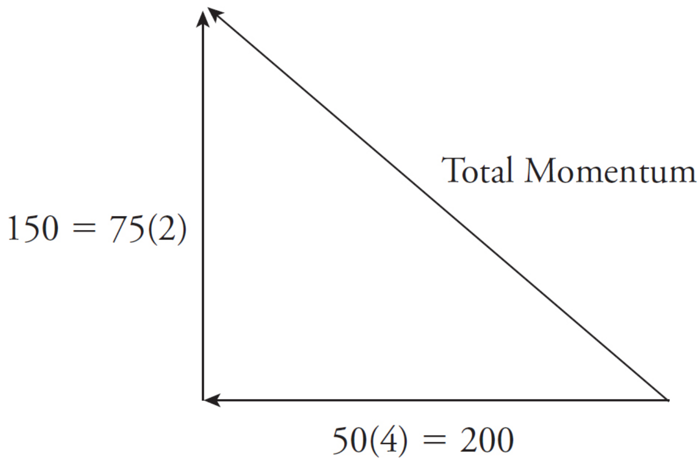
21. **A**

For a spring that is not linear (i.e., does not obey Hooke's law) the energy stored is not $\frac{1}{2}kx^2$. The magnitude of the energy stored will be equal to the magnitude of the work done to stretch the spring to x_1. The steps to calculate the work are shown below.

$$W = \int F\,dx$$
$$W = \int_0^{x_1} (kx^{1.5})dx$$
$$W = \left.\frac{kx^{2.5}}{2.5}\right|_0^{x_1}$$
$$W = \frac{2kx_1^{\ 2.5}}{5}$$

22. **D**

Applying the Law of Conservation of Linear Momentum tells us that the total momentum of the system after the collision is equal to the total momentum of the system before the collision. Momentum can be calculated by multiplying the mass times the velocity for each skater and adding the vectors as shown below. You can use the Pythagorean theorem to solve for the total momentum, or realize this is a 3-4-5 right triangle, so the total momentum is equal to 250 kg·m/s.



23. **D**

When objects stick together, a perfectly inelastic collision has occurred. For this situation, kinetic energy is not conserved. This eliminates (B), (C), and (E). Linear momentum is conserved when the net force on the system is zero, so only (D) combines all this information correctly.

24. **C**

Acceleration is the second derivative of the position. Take the derivative of the position function twice and then plug in t = 1.5 s to get the correct answer, as shown below.

$$\frac{dx}{dt} = 6t^2 + 4$$
$$\frac{d^2x}{dt^2} = 12t$$
$$\frac{d^2x}{dt^2} = 12(1.5) = 18$$

25. **C**

Use the Law of Conservation of Linear Momentum to determine how quickly she'll move after throwing the wrench. We know the total momentum of the system must be 0, so

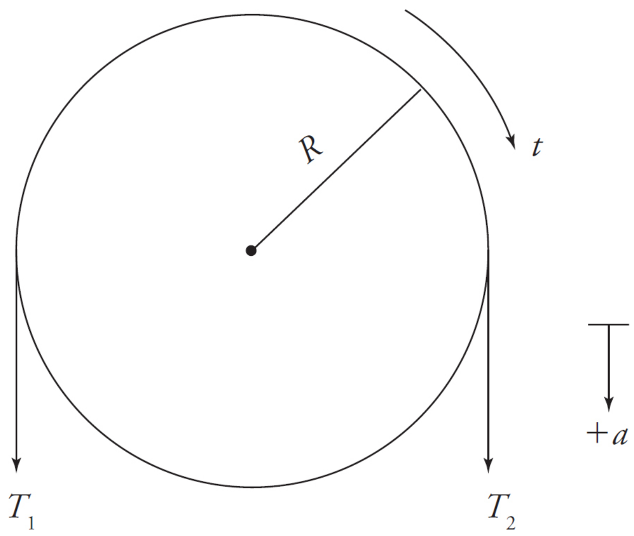
$$0 = m_1v_1 + m_2v_2$$
$$m_1v_1 = m_2v_2$$
$$v_1 = m_2v_2/m_1$$
$$= [(2\text{ kg})(30\text{ m/s})]/(60\text{ kg}) = 1\text{ m/s}$$

If she moves at 1 m/s, it will take 35 s to get back to the shuttle.

26. **C**

The pulley will rotate because there is a net torque on the pulley due to tension 1 and tension 2. Apply Newton's Second Law for rotation to the diagram below and then substitute $\alpha = \frac{a}{r}$. Take clockwise and downward to be positive, and remember that torque is the cross product of force and distance.

$$\sum \tau = I\alpha$$
$$(T_2 - T_1)R = I\frac{a}{R}$$



27. **B**

The force of gravity acting between the masses can be calculated using Newton's Law of Gravity: $F = \frac{Gm_1m_2}{r^2}$. When the test mass is outside of the sphere it is an inverse square relationship, so (C), (D), and (E) can be eliminated. When the test mass is inside the sphere, at a radius $r < R$, the sphere still attracts the test mass with amount of mass, M', that is at a radius less than the test mass. M' is proportional to the total mass by the volume contained compared to the total volume as shown below.

$$\frac{M'}{M} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$$

$$M' = M\frac{r^3}{R^3}$$

This produces a gravitational force that is linear for the region $r < R$, as shown below.

$$F = \frac{GmM'}{r^2}$$

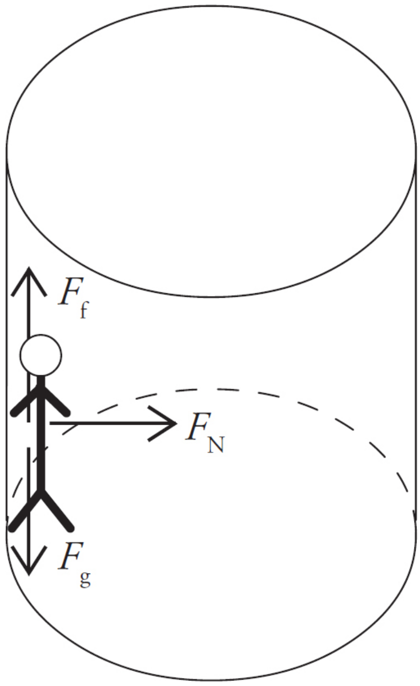
$$F = \frac{Gm\left(M\frac{r^3}{R^3}\right)}{r^2}$$

$$F = \frac{GmMr}{R^3}$$

Therefore, (A) can be eliminated. This can also be found conceptually by realizing that the net gravitational force will only be zero where $r = 0$.

28. **E**

First, draw a free-body diagram:



In order to stay in place, the force of friction needs to have a magnitude equal to the force of gravity's.

$$F_g = F_f$$
$$mg = \mu F_N$$

Next, we need to solve for the normal force. We can do this because the person is undergoing uniform circular motion, so we can set the normal force equal to centripetal force.

$$F_N = F_C$$

$$F_N = mv^2/r$$

Finally, substitute this value into the previous result:

$$mg = \mu(mv^2/r)$$
$$g = \mu v^2/r$$

We see that mass actually drops out of the equation. The only variables that matter are the coefficient of friction, speed, and the radius.

29. **B**

The formula for frequency of a pendulum is

$$f_{\text{pendulum}} = \frac{1}{2\pi}\sqrt{(g/L)}$$

This means that the frequency is proportional to the square root of gravity. Therefore, since the moon's gravity is $\frac{1}{6}$ of Earth's, the new frequency will be $\left(\frac{1}{6}\right)^{\frac{1}{2}}$ times the old frequency.

30. **B**

Power is equal to the rate work is done and also the dot product of force and velocity.

$$P = Fv$$
$$400\text{ W} = F(20\text{ m/s})$$
$$F = 20\text{ N}$$

31. **C**

The energy of the oscillating spring-mass system will remain constant. When it is pulled down 10 cm the energy will be stored in the spring, and when it passes the equilibrium position, all of the energy will be kinetic energy.

$$E_1 = E_2$$
$$\frac{1}{2}kx^2 = K$$
$$\frac{1}{2}(800\ \frac{\text{N}}{\text{m}})(0.1\text{ m})^2 = K$$
$$4\text{ J} = K$$

32. **D**

The constant kinematics equations ignore the air resistance that decreases the total mechanical energy of the ball as it falls. The force due to air resistance also increases the faster the ball is going, so the force is increasing with time. This would make the acceleration of the ball begin at 9.8 m/s^2 and then decrease as the ball falls. This eliminates (C). The ball is rigid so it will not change shape when falling, which eliminates (A). The acceleration of gravity will be constant over the 6 meters that the ball falls, so this eliminates (B) and (E). Therefore, (D) is left and is correct because the fall is speeding up at a decreasing rate of acceleration.

33. **C**

The acceleration of the object will be due to gravity and air resistance. Air resistance is proportional to the velocity of the object. Since air resistance decreases the total mechanical energy of the object, the greatest velocity will occur at the point closest to the initial launch, point A. The greatest acceleration will occur when the air resistance and gravity are in the same direction and when the object is traveling the fastest. This is also at point A. Therefore, (C) is correct.

34. **B**

The acceleration of the center of mass is zero because the disk is rolling at a constant velocity. The contact point, *P*, is instantaneously at rest and then moves upward, which implies the acceleration is upward. Combining these two pieces of information indicates (B) is the correct answer.

35. **E**

The escape velocity for a planet can be determined by making the total energy (kinetic energy plus gravitational potential energy) equal to zero. This implies that the spaceship has escaped from the gravitational pull of the planet (i.e., reached infinity) with no more velocity. This calculation is shown below.

$$\frac{1}{2}m_1 v_{esc}^2 - \frac{Gm_1 m_2}{r} = 0$$

$$v_{esc}^2 = \frac{2Gm_2}{r}$$

$$v_{esc} = \sqrt{\frac{2Gm_2}{r}}$$

Examining the last equation, one can see that the effects of doubling the mass and doubling the radius will cancel each other out. Therefore, the escape velocity will remain v_0.

SECTION II, MECHANICS

1. (a) Use Conservation of Energy since all of the energy stored in the spring will be transferred into kinetic energy of block *A*. Realize the compression of the spring is given as *d*.

$$E_1 = E_2$$

$$\frac{1}{2}kd^2 = \frac{1}{2}Mv^2$$

$$\frac{kd^2}{M} = v^2$$

$$v = d\sqrt{\frac{k}{M}}$$

(b) Use Conservation of Linear Momentum for the collision. During a perfectly inelastic collision the two blocks stick together and continue at the same velocity.

$$p_1 = p_2$$

$$Md\sqrt{\frac{k}{M}} = 2Mv_2$$

$$v_2 = \frac{d}{2}\sqrt{\frac{k}{M}}$$

(c) The change in the kinetic energy during the collision is given by $\Delta K = K_2 - K_1$.

$$\Delta K = K_2 - K_1$$

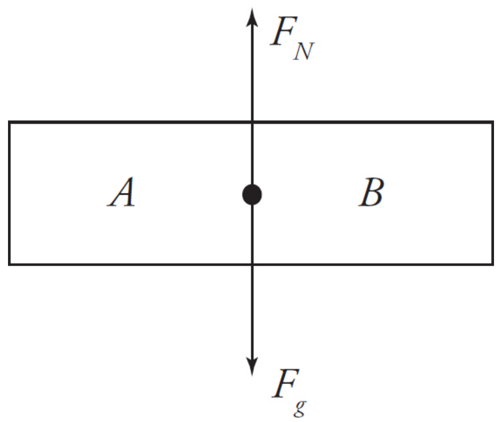
$$\Delta K = \frac{1}{2}(2M)v_2^{\,2} - \frac{1}{2}(M)v_1^{\,2}$$

$$\Delta K = \frac{1}{2}(2M)\left(\frac{d}{2}\sqrt{\frac{k}{M}}\right)^2 - \frac{1}{2}(M)\left(d\sqrt{\frac{k}{M}}\right)^2$$

$$\Delta K = \frac{d^2k}{4} - \frac{d^2k}{2}$$

$$\Delta K = -\frac{d^2k}{4}$$

(d) (See the diagram on the next page for more details.) The blocks are traveling along a curved path and at the top of the hill they are traveling with the same velocity as in block B since it is at the same elevation. They are also traveling in circular motion at the top of the hill. The gravitational force is pointing toward the center of the circle, so it will be positive and the normal force is pointing away from the center of the circle, so it will be negative.



$$\sum F_c = \frac{mv^2}{r}$$

$$F_g - F_N = \frac{(2M)\left(\frac{d}{2}\sqrt{\frac{k}{M}}\right)^2}{R}$$

$$(2M)g - \frac{(2M)\left(\frac{d^2k}{4M}\right)}{R} = F_N$$

$$2Mg - \frac{d^2k}{2R} = F_N$$

2. (a) The blocks will collide when the total distance traveled by the blocks is 1 m.

$$d_1 + d_2 = 1 \text{ m} \rightarrow (\tfrac{1}{2}gt^2) + (v_{2,0}t - \tfrac{1}{2}gt^2) = 1 \text{ m} \rightarrow v_{2,0}t = 1 \text{ m} \rightarrow t = \frac{1\text{ m}}{v_{2,0}} = \frac{1\text{ m}}{10\text{ m/s}} = 0.1 \text{ s}$$

(b) The height of the collision will be equal to d_2.

$$d_2 = v_{2,0}t - \frac{1}{2}gt^2 = (10\text{ m/s})(0.1\text{ s}) - (\tfrac{1}{2})(10\text{ m/s}^2)(0.1)^2 = 0.95\text{ m}$$

(c) We can use one of our Big 5 equations to calculate the collision speeds of the blocks, $v_{1,c}$and $v_{2,c}$.

$$v_{1,c} = v_{1,0} + at = (0 \text{ m/s}) + (10 \text{ m/s}^2)(0.1 \text{ s}) = 1 \text{ m/s}$$

$$v_{2,c} = v_{2,0} - at = (10 \text{ m/s}) - (10 \text{ m/s}^2)(0.1 \text{ s}) = 9 \text{ m/s}$$

Note the changing sign for the acceleration terms above. This is because gravity increases the speed of the falling block but decreases the speed of the launched block.

(d) First, solve for the speed of the blocks after the collision by using the equation for conservation of momentum in a perfectly inelastic collision.

$$m_1\mathbf{v}_{1,c} + m_2\mathbf{v}_{2,c} = (m_1 + m_2)\mathbf{v}_f \rightarrow \mathbf{v}_f$$

$$= \frac{m_1\mathbf{v}_{1,c} + m_2\mathbf{v}_{2,c}}{m_1 + m_2}$$

$$= \frac{(10 \text{ kg})(-1 \text{ m/s}) + (10 \text{ kg})(9 \text{ m/s})}{(10 \text{ kg} + 10 \text{ kg})} = 4 \text{ m/s}$$

Now use one of the Big 5 to solve for maximum height, using 4 m/s as the initial velocity and 0 m/s (the velocity at the peak) as the final.

$$v_f^2 = v_0^2 + 2ad \rightarrow d = \frac{\left(v_f^2 - v_0^2\right)}{2a} = \frac{(0 \text{ m/s})^2 - (4 \text{ m/s})^2}{2\left(-10 \text{ m/s}^2\right)} = 0.8 \text{ m}$$

Finally, don't forget to add this to the height at which the collision occurred (0.95 m) for a final height of 1.75 m.

(e) First, solve for the launch speed in terms of the time of collision. From our work in (a), we know

$$d_1 + d_2 = (\tfrac{1}{2} gt^2) + (v_{2,0}t - \tfrac{1}{2} gt^2) = v_{2,0}t = 1 \text{ m} \rightarrow v_{2,0} = \frac{1 \text{ m}}{t} \text{ or, alternatively, } t = \frac{1 \text{ m}}{v_{2,0}}$$

Next, solve for the collision speeds as done in (c).

$$v_{1,c} = v_{1,0} + at = v_{1,0} + g\left(\frac{1}{v_{2,0}}\right) = \frac{10 \text{ m/s}^2}{v_{2,0}}$$

$$v_{2,c} = v_{2,0} + at = v_{2,0} - g\left(\frac{1}{v_{2,0}}\right) = v_{2,0} - \frac{10 \text{ m/s}^2}{v_{2,0}}$$

Using these, calculate the post-collision speed as done in (d).

$$m_1\mathbf{v}_{1,c} + m_2\mathbf{v}_{2,c} = (m_1 + m_2)\mathbf{v}_f \rightarrow \mathbf{v}_f$$

$$= \frac{m_1\mathbf{v}_{1,c} + m_2\mathbf{v}_{2,c}}{m_1 + m_2}$$

$$= \frac{-(10\,\text{kg})\left(\frac{10\ \text{m/s}^2}{v_{2,0}} + 10\,\text{kg}\left(v_{2,0} - \frac{10\ \text{m/s}^2}{v_{2,0}}\right)\right)}{10\,\text{kg} + 10\,\text{kg}}$$

$$= \frac{v_{2,0} - \frac{20\ \text{m/s}^2}{v_{2,0}}}{2\ \text{kg}}$$

After the collision, the blocks will have to travel a distance equal to d_1 in order to reach 1 m height. The final step is to setup a kinematics equation using the last result as v_0, $d_1$as the distance to travel, 0 m/s as v_f (since we want it to peak at this point), and gravity as the acceleration.

$$v_f^2 = v_0^2 + 2ad \rightarrow (0\ \text{m/s})^2$$

$$= \left(\frac{v_{2,0} - \frac{20\ \text{m/s}^2}{v_{2,0}}}{2\ \text{kg}}\right)^2 + 2\left(-10\ \text{m/s}^2\right)\left(\frac{\frac{1}{2}\left(10\ \text{m/s}^2\right)1\,\text{m}}{\left(v_{2,0}\right)^2}\right) \rightarrow v_{2,0}$$

$$= 2\sqrt{10\ \text{m/s}}$$

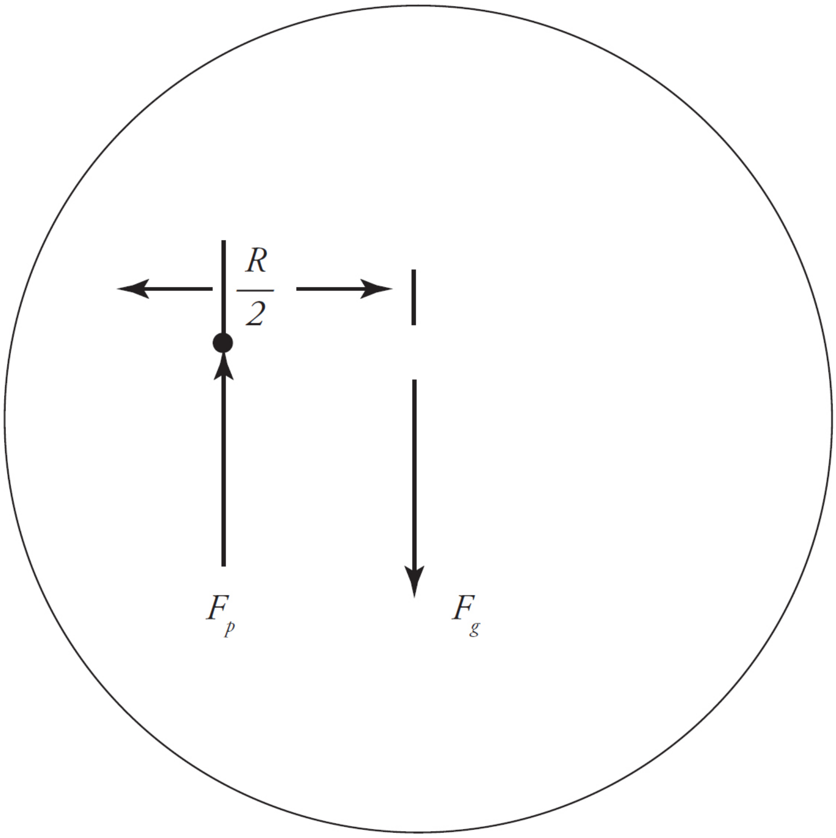
3. (a) Use the parallel axis theorem to determine the rotational inertia of the object about the pin.

$$I = I_{\text{cm}} + md^2$$

$$I = \frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2$$

$$I = \frac{3}{4}MR^2$$

(b) When the object is first released, gravity provides a torque about the pin causing the disk to rotate about the pin. A free-body diagram for the disk is shown below.



Use Newton's Second Law of Rotation about the pin to determine the angular acceleration.

$$\sum \tau_{pin} = I\alpha$$
$$Mg\left(\frac{R}{2}\right) = \left(\frac{3MR^2}{4}\right)\alpha$$
$$\frac{2g}{3R} = \alpha$$

(c) Use the Law of Conservation of Mechanical Energy to determine the angular velocity at the vertical position. All the initial gravitational potential energy is converted into rotational kinetic energy. The center of mass falls half the distance of the radius.

$$E_1 = E_2$$
$$Mg\left(\frac{R}{2}\right) = \frac{1}{2}I\omega^2$$
$$\frac{MgR}{2} = \frac{1}{2}\left(\frac{3MR^2}{4}\right)\omega^2$$
$$\frac{4g}{3R} = \omega^2$$
$$\sqrt{\frac{4g}{3R}} = \omega$$

(d) Now the disk is undergoing simple harmonic motion because it is undergoing small angle oscillations. Derive a differential equation of the form $\frac{d^2x}{dt^2} = -\omega^2 x$ to determine ω. Start with Newton's Second Law of Rotation to end up with a differential equation because $\alpha = \frac{d^2\theta}{dt^2}$.

$$\sum \tau_{pin} = I\alpha$$

$$-Mg\sin\theta\left(\frac{R}{2}\right) = \left(\frac{3MR^2}{4}\right)\alpha.$$

Using $\sin\theta \approx \theta$, for small θ,

$$-\frac{2g}{3R}\theta = \frac{d^2\theta}{dt^2}.$$

Therefore, $\omega = \sqrt{\frac{2g}{3R}}$.