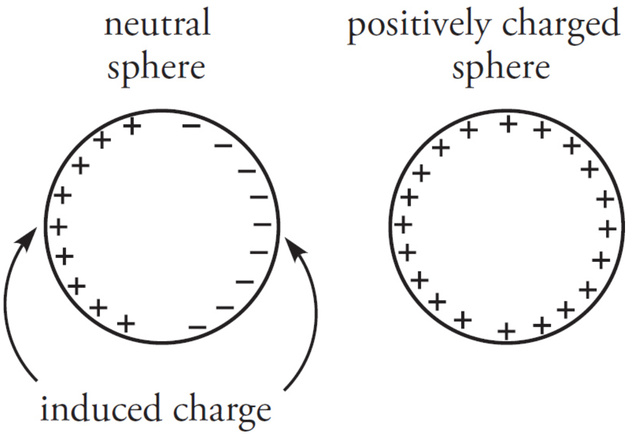
SECTION I, ELECTRICITY AND MAGNETISM

36. **C**

The proximity of the charged sphere will induce negative charge to move to the side of the uncharged sphere closer to the charged sphere. Since the induced negative charge is closer than the induced positive charge to the charged sphere, there will be a net electrostatic attraction between the spheres.



37. **E**

Typically, the formula for power dissipated by a circuit is expressed as $P = IV$. However, it can also be written as $P = I^2R$ or $P = V^2/R$, which would appear to give contradictory answers to this question. The correct answer is that it cannot be determined because there's no way to double the resistance of a circuit without affecting anything else. Ohm's Law tells us that $V = IR$, so we would have to know whether the voltage or current is changing as a result of the doubled resistance before we can address the change to the power dissipated.

38. **E**

When the particle enters the magnetic field, the magnetic force provides the centripetal force to cause the particle to execute uniform circular motion:

$$|q|vB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{|q|B}$$

Since v and B are the same for all the particles, the largest r is found by maximizing the ratio $m/|q|$. Furthermore, since the ratio of u/e (atomic mass unit/magnitude of elementary charge) is a constant, the answer depends only on the ratio of "factor of u" to "factor of charge." The ratio 20/1 is largest, so it will have the largest r.

39. **A**

The only place with 0 net electric field would be in the center of the rectangle. Moving up along Line A would give a net electric field pointing to the right. This is because the top two charges, which would both force a positive particle to the right (in addition to vertical effects), are closer and therefore more impactful than the bottom two charges, which would oppose them by trying to make a positive particle move to the left. Similarly, moving down along Line A would give a net electric field to the left. Moving to the right along Line B would give a net electric field upward. Moving to the left along Line B would give a net electric field downward.

40. **B**

The resistance of a wire made of a material with resistivity ρ and with length L and cross-sectional area A is given by the equation $R = \rho L/A$. Since Wire B has the greatest length and smallest cross-sectional area, it has the greatest resistance.

41. **E**

From the equation $F = qvB$, we see that 1 tesla is equal to 1 N·s/(C·m). Since

$$\frac{1\,\mathrm{N\cdot s}}{\mathrm{C\cdot m}} = \frac{1\,\mathrm{N}}{\mathrm{A\cdot m}} = \frac{1\,\mathrm{J}}{\mathrm{A\cdot m^2}}$$

(A) and (C) are eliminated.

Furthermore,

$$\frac{1\,\mathrm{N\cdot s}}{\mathrm{C\cdot m}} = \frac{1\,\mathrm{kg\cdot m/s}}{\mathrm{C\cdot m}} = \frac{1\,\mathrm{kg}}{\mathrm{C\cdot s}}$$

eliminating (B). Finally, since

$$\frac{1\,\mathrm{J}}{\mathrm{A\cdot m^2}} = \frac{1\,\mathrm{C\cdot V}}{(\mathrm{C/s})\cdot \mathrm{m^2}} = \frac{1\,\mathrm{V\cdot s}}{\mathrm{m^2}}$$

(D) is also eliminated.

42. **C**

Gauss's law states that the total electric flux through a closed surface is equal to $(1/\varepsilon_0)$ times the net charge enclosed by the surface. Both the cube and the sphere contain the same net charge, so Φ_C must be equal to Φ_S.

43. **E**

The particle would experience oscillations between the positions x = 30 cm and x = 70 cm. At the position x = 50 cm, it would have no net force acting on it, but this would be similar to a spring having no net force as it passes through the equilibrium position. It will continue moving until an opposing force reduces its speed to 0, which would occur at x = 30 cm. It would then be pushed to right until it reached x = 70 cm, and the cycle would repeat infinitely.

44. **A**

The Table of Information provides the following conversion factor: 1 e = 1.6×10^{-19} C. Multiplying the magnitude of charge, 1 C, by this conversion factor gives the following:

$$1\,\mathrm{C} = \frac{-1\,e}{1.60\times 10^{-19}\,C} = 6.25\times 10^{18}\,e$$

There are 6.25×10^{18} electrons in 1 coulomb of negative charge.

45. **E**

Resistors R_2, R_3, and R_4 are in parallel, so their equivalent resistance, $R_{2\text{-}3\text{-}4}$, satisfies

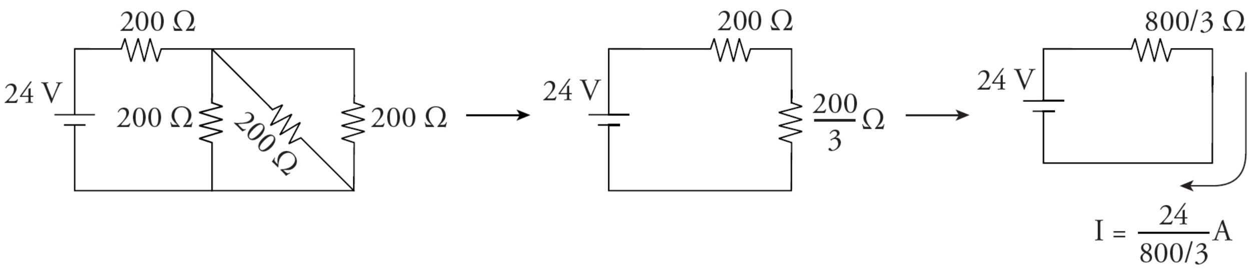
$$\frac{1}{R_{2-3-4}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{3}{200\ \Omega} \Rightarrow R_{2-3-4} = \frac{200}{3}\ \Omega$$

Since this is in series with R_1, the total resistance in the circuit is

$$R = R_1 + R_{2-3-4} = \frac{800}{3}\,\Omega$$

The current provided by the source must therefore be

$$I = \frac{\varepsilon}{R} = \frac{24\,V}{\frac{800}{3}\,\Omega} = \frac{9}{100}\,\text{A} = \frac{90}{1000}\,\text{A} = 90\text{ mA}$$



Drawing a series of equivalent circuits can make it easier to see what the resistance should be at each step.

46. **E**

The current through R_4 is $\frac{1}{3}$ the current through R_1, and $R_4 = R_1$, so

$$\frac{P_1}{P_4} = \frac{I_1^2 R_1}{I_4^2 R_4} = \frac{I_1^2 R_1}{\left(\frac{1}{3} I_1\right) R_1} = 9$$

47. **D**

Note that all answers have the same number, so for this problem, don't even worry about multiplying the numbers—just focus on the units. Because the time constant for an RC circuit is equal to the product of resistance and capacitance, $\tau = RC$, this product has the dimensions of time. If you don't know that off the top of your head, just be sure to carefully break the farad and ohm into their subunits, simplifying where possible:

1 F × 1 Ω = 1 C/V × 1 V/A = 1 C/A = 1 s

48. **B**

Apply Faraday's Law of Electromagnetic Induction:

$$\begin{aligned}
\oint E \cdot d\ell &= -\frac{d\Phi_B}{dt} \\
E(2\pi r) &= -\frac{d}{dt}(BA) \\
&= -A\frac{dB}{dt} \\
&= -\pi r^2 \cdot \frac{d}{dt}(2t - 2t^2) \\
&= -\pi r^2 (2 - 4t) \\
&= -r(1 - 2t) \\
E &= r(2t - 1)
\end{aligned}$$

Since r = 1 m, the value of E at t = 1 s is E = 1 N/C.

49. **A**

The magnetic force, $\mathbf{F}_B$, on the top horizontal wire of the loop must be directed upward and have magnitude mg. (The magnetic forces on the vertical portions of the wire will cancel.) By the right-hand rule, the current in the top horizontal wire must be directed to the right—because **B** is directed into the plane of the page—in order for $\mathbf{F}_B$ to be directed upward; therefore, the current in the loop must be clockwise, eliminating (B) and (D). Since $F_B = IxB$, we must have

$$IxB = mg \Rightarrow I = \frac{mg}{xB}$$

50. **C**

First, find the difference in potential between the starting location (2 cm away) and the location in question (1 cm away):

$$\begin{aligned}\Delta\Phi &= \Phi_f - \Phi_0 \\ &= \frac{kQ}{r_f} - \frac{kQ}{r_0} \\ &= \frac{(9\times10^9)(10\times10^{-9})}{1\times10^{-2}} - \frac{(9\times10^9)(10\times10^{-9})}{2\times10^{-2}} \\ &= 4500 \text{ V}\end{aligned}$$

Next, find the change in potential energy.

$$\Delta PE = q\Delta\Phi = (-5\times10^{-9})(4500) = -2.25\times10^{-5} \text{ J}$$

Because total energy will be conserved, the change in kinetic energy must be 2.25 x 10^{-5}J. This can be used to find the final speed.

$$\begin{aligned}\Delta KE = KE_f - KE_0 &\rightarrow 2.25\times10^{-5} = \frac{1}{2}mv_f^2 \rightarrow v_f \\ &= \sqrt{\frac{2(2.25\times10^{-5})}{5\times10^{-10}}} \\ &= 300 \text{ m/s}\end{aligned}$$

51. **A**

Call the top wire (the one carrying a current I to the right) Wire 1, and call the bottom wire (carrying a current $2I$ to the left) Wire 2. Then in the region between the wires, the individual magnetic field vectors due to the wires are both directed into the plane of the page, so they could not cancel in this region. Therefore, the total magnetic field could not be zero at either Point 2 or Point 3. This eliminates (B), (C), (D), and (E), so the answer must be (A). Since the magnetic field created by a current-carrying wire is proportional to the current and inversely proportional to the distance from the wire, the fact that Point 1 is in a region where the individual magnetic field vectors created by the wires point in opposite directions and that Point 1 is twice as far from Wire 2 as from Wire 1 implies that the total magnetic field there will be zero.

52. **A**

If the mass of the $+q$ charge is m, then its acceleration is

$$a = \frac{F_E}{m} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{mr^2}$$

Because a should decrease as r increases, you can eliminate (C) and (E). The graph in (A) best depicts an inverse-square relationship between a and r.

53. **C**

Once the switch is closed, the capacitors are in parallel, which means the voltage across C_1 must equal the voltage across C_2. Since $V = Q/C$,

$$\frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \Rightarrow Q_2' = \frac{C_2}{C_1}Q_1' = \frac{6\mu\text{F}}{3\mu\text{F}}Q_1' = 2Q_1'$$

This makes sense qualitatively: C_2 has twice the capacitance of C_1, so it should hold twice the charge. The total charge, 24 μC + 12 μC = 36 μC, must be redistributed so that $Q_2' = 2Q_1'$. Therefore, we see that $Q_2' = 24$ μC (and $Q_1' = 12$ μC).

54. **E**

Because **v** is parallel to **B**, the charges in the bar feel no magnetic force, so there will be no movement of charge in the bar and no motional emf.

55. **E**

Along the line joining the two charges of an electric dipole, the individual electric field vectors point in the same direction, so they add constructively. At the point equidistant from both charges, the total electric field vector has magnitude

$$E_+ + E_- = 2E = 2\frac{kQ}{\left(\frac{1}{2}d\right)^2} = \frac{8kQ}{d^2}$$

$$= \frac{8(9\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4.0\times10^{-9}\,\text{C})}{(2.0\times10^{-2}\,\text{m}^2)^2}$$

$$= 72\times10^4\ \text{N/C}$$

$$= 7.2\times10^5\ \text{N/C (or V/m)}$$

56. **E**

Start with basic dimensional analysis. Choices (A) and (B) can be eliminated because those expressions do not yield units of capacitance. Imagine placing equal but opposite charges on the spheres, say, $+Q$ on the inner sphere and $-Q$ on the outer sphere. Then the electric field between the spheres is radial and depends only on $+Q$, and its strength is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Therefore, the potential difference between the spheres is

$$V = \int_R^{2R} E\cdot d\mathbf{r} = \frac{Q}{4\pi\varepsilon_0}\int_R^{2R}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)_R^{2R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{2R}\right) = \frac{Q}{8\pi\varepsilon_0 R}$$

Now, by definition, $C = Q/V$, so

$$C = \frac{Q}{V} = Q \cdot \frac{1}{V} = Q \cdot \frac{8\pi\varepsilon_0 R}{Q} = 8\pi\varepsilon_0 R$$

57. **C**

Since the magnetic force is always perpendicular to the object's velocity, it does zero work on any charged particle. Zero work means zero change in kinetic energy, so the speed remains the same. Remember, the magnetic force can only change the direction of a charged particle's velocity, not its speed.

58. **B**

The presence of the inductor in the rightmost branch effectively removes that branch from the circuit at time $t = 0$ (the inductor produces a large back emf when the switch is closed and the current jumps abruptly). Initially, then, current only flows in the loop containing the resistors r and R. Since their total resistance is $r + R$, the initial current is $\mathcal{E}/(r + R)$.

59. **B**

The correct answer should have dimensions of energy. Using the equation $U_L = \frac{1}{2} LI^2$ from the Table of Information, L should have units of J/A^2. Choices (A), (B), (C), and (E) all have units of $J/A^2 \times V^2/\Omega^2 = J/A^2 \times A^2 = J$, which is a unit of energy. Choice (D) has an extra factor of resistance in the numerator, and can be eliminated. After a long time, the current through the branch containing the inductor increases to its maximum value. With all three resistors in play, the total resistance is $R_{eq} = r + \frac{R}{2}$, since $\frac{R}{2}$ is the equivalent resistance of the two resistors in parallel. The current provided by the source, $\frac{\mathcal{E}}{R_{eq}}$, splits at the junction leading to the parallel combination. The amount which flows through the inductor is half the total.

$$I = \frac{1}{2} \cdot \frac{\mathcal{E}}{\left(r + \frac{R}{2}\right)} = \frac{\mathcal{E}}{2r + R}$$

so the energy stored in the inductor is

$$U_L = \frac{LI^2}{2} = \frac{L}{2}\left(\frac{\mathcal{E}}{(2r+R)}\right)^2 = \frac{L\mathcal{E}^2}{2(2r+R)^2}$$

60. **A**

Once the switch is opened, the resistor r is cut off from the circuit, so no current passes through it. (Current *will* flow around the loop containing the resistors labeled R, gradually decreasing until the energy stored in the inductor is exhausted.)

61. **C**

If a conducting sphere contains a charge of $+q$ within an inner cavity, a charge of $-q$ will move to the wall of the cavity to "guard" the interior of the sphere from an electrostatic field, regardless of the size, shape, or location of the cavity. As a result, a charge of $+q$ is left on the exterior of the sphere (and it will be uniform). So, at points outside the sphere, the sphere behaves as if this charge $+q$ were concentrated at its center, so the electric field outside is simply kQ/r^2. Since points X and Y are at the same distance from the center of the sphere, the electric field strength at Y will be the same as at X.

62. **B**

A particle of negative charge is attracted to the two positive charges in the problem, and will gain kinetic energy as it approaches. By the work-energy theorem, the electric field must be doing positive work on the particle. Alternatively, the particle is attracted to the two charges it's moving toward, which means $\mathbf{F}_E$ and $\mathbf{d}$ are in the same direction, so the work done by $\mathbf{E}$ is positive. Eliminate (C) and (E).

To decide between the remaining choices, first determine the electric potential at Point P. Realize that Point P is 5 meters away from each charge because it is a 3-4-5 right triangle.

$$V = \sum \frac{kQ}{r}$$

$$V_P = \frac{kQ}{5} + \frac{kQ}{5} = \frac{2kQ}{5}$$

The work done by the electric field will be positive because a $-Q$ charge would be attracted to point P by the two positive Q charges. Also the work done by the force is the negative of the change in potential energy.

$$\Delta V = \frac{\Delta U}{q}$$

$$V_P - V_\infty = \frac{\Delta U}{-Q}$$

$$\frac{2kQ}{5} - 0 = \frac{\Delta U}{-Q}$$

$$\Delta U = \frac{-2kQ^2}{5}$$

$$W = -\Delta U = \frac{2kQ^2}{5}$$

63. **E**

The current in an L–R series circuit—in which initially no current flows—increases with time t according to the equation

$$I(t) = I_{max}(1 - e^{-t/\tau})$$

where $I_{max} = \mathcal{E}/R$ and τ is the inductive time constant, L/R. The time t at which I = (99%)I_{max} is found as follows:

$$1 - e^{-t/\tau} = \frac{99}{100}$$

$$e^{-t/\tau} = \frac{1}{100}$$

$$-t/\tau = \ln \frac{1}{100}$$

$$t = (-\ln \frac{1}{100})\tau = (\ln 100)\tau = (\ln 100)L/R$$

Alternatively, the solution to this question can be found by using process of elimination. Choice (B) can be eliminated because it does not incorporate the 99%. Choices (A) and (C) can be eliminated because they give negative results. Choice (D) can be eliminated because ln(100/99) is close to zero. This leaves only (E), which satisfies the given conditions.

64. **E**

First solve for the resistance of each lightbulb as shown below:

$$P = V^2/R$$

$$R = V^2/P = (120\text{ V})^2/(40\text{ W}) = 360\ \Omega$$

Next solve for the equivalent resistance, R_{eq}, of the circuit as follows:

$$R_{eq} = V/I = (120\text{ V})/(5\text{ A}) = 24\ \Omega$$

If you used a current less than 5 A (the maximum allowable before the fuse blows), then the R_{eq} would increase. Finally, solve for the number of 360 Ω resistors (lightbulbs) that can be placed in a parallel circuit with an equivalent resistance of 24 Ω.

$$1/R_{eq} = n/R$$

$$n = R/R_{eq} = (360\ \Omega)/(24\ \Omega) = 15$$

Thus, a maximum of 15 lightbulbs can be connected in parallel with the 120 V source.

65. **D**

Since the magnetic flux out of the page is decreasing, the induced current will oppose this change (as always), attempting to create more magnetic flux out of the page. In order to do this, the current must circulate in the counterclockwise direction (remember the right-hand rule). Eliminate (A) and (B). As B decreases *uniformly* (that is, while dB/dt is negative and *constant*), the induced emf,

$$\mathcal{E} = \frac{d\Phi_B}{dt} = -A\frac{dB}{dt}$$

is nonzero and constant, which implies that the induced current, $I = \mathcal{E}/R$, is also nonzero and constant.

66. **D**

Treat the configuration as three capacitors in series, each with a distance between the plates of $\frac{d}{3}$. The capacitance of the two vacuum capacitors will be $\frac{\varepsilon_o A}{\left(\frac{d}{3}\right)} = \frac{3\varepsilon_o A}{d}$, and the capacitance of the dielectric capacitor will be that value times K. Now solve for the total capacitance for the three capacitors in series.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{\left(\frac{3\varepsilon_o A}{d}\right)} + \frac{1}{\left(\frac{3K\varepsilon_o A}{d}\right)} + \frac{1}{\left(\frac{3\varepsilon_o A}{d}\right)}$$

$$\frac{1}{C_{eq}} = \frac{d}{3\varepsilon_o A} + \frac{d}{3K\varepsilon_o A} + \frac{d}{3\varepsilon_o A}$$

$$\frac{1}{C_{eq}} = \frac{2Kd + d}{3K\varepsilon_o A}$$

$$C_{eq} = \frac{3K\varepsilon_o A}{d(2K+1)}$$

Alternatively, this problem can be solved by process of elimination. First, use dimensional analysis. The Table of Information provided with the AP Test notes that capacitance should have units consistent with $\varepsilon_0 A/d$; (B) fails this test. Second, pretend that the dielectric constant $K = 1$. This is the same as not having a dielectric, so plugging in $K = 1$ should give $C = \varepsilon_0 A/d$. Eliminate (A) and (E). Finally, as K increases, the capacitor becomes stronger, so C should increase. Once (C) fails this test, only (D) is left.

67. **A**

The potential is zero at the point midway between the charges, but nowhere is the electric field equal to zero (except at infinity).

68. **D**

Ampere's law states that

$$\oint B \cdot d\ell = \mu_0 I_{\text{net through loop}}$$

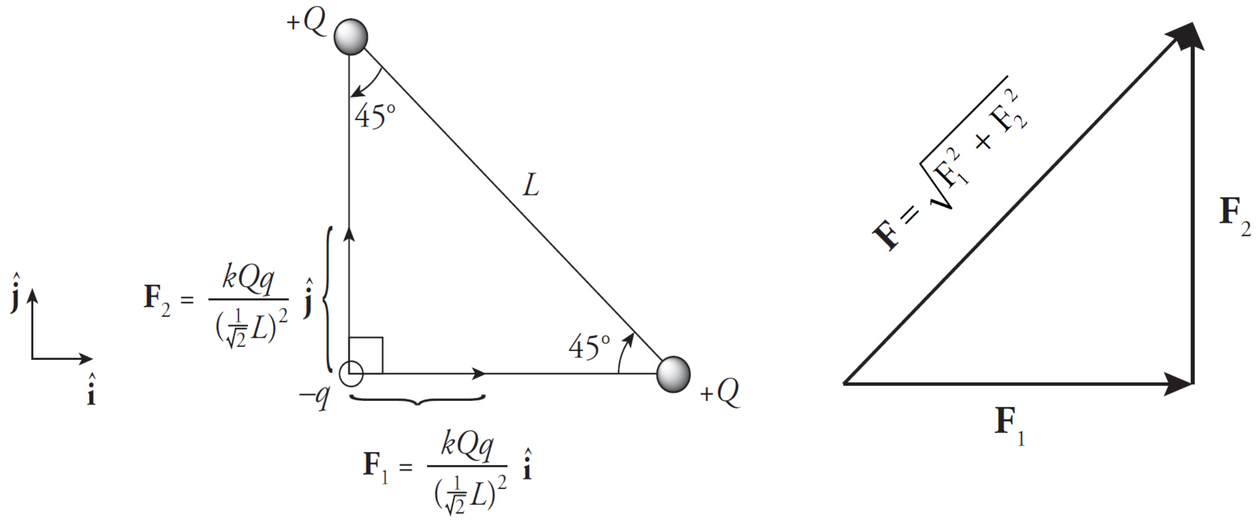
Because a total current of $2I + 4I = 6I$ passes through the interior of the loop in one direction, and a total current of $I + 3I = 4I$ passes through in the opposite direction, the net current passing through the loop is $6I - 4I = 2I$. Therefore, the absolute value of the integral of **B** around the loop WXYZ is equal to $\mu_0(2I)$, which is equivalent to (D).

69. **B**

Because $\Delta U = q(\Delta V)$, our answer must be negative. ΔV will be the same in both cases, and the sign on q changed, so the sign on ΔU has to change as well. Eliminate (C) and (D). Next, notice that the formula for ΔU does not depend on the distance traveled. That means the different path will not impact the magnitude of ΔU. Eliminate (A). Finally, knowing the values of Q_1 and Q_2 would be important if we wanted to calculate a numerical answer, but that knowledge is unnecessary for determining the relationship in the question.

70. **A**

First, note that the distance between $-q$ and each charge $+Q$ is $\frac{1}{\sqrt{2}}L$. Now, refer to the following diagram, where we've invoked Coulomb's law to determine the two electrostatic forces:



We find that the magnitude of **F** is

$$F = \frac{kQq}{\left(\frac{1}{\sqrt{2}}L\right)^2}\sqrt{1^2+1^2} = k\frac{2\sqrt{2}\cdot Qq}{L^2} = \frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}\cdot Qq}{L^2} = \frac{\sqrt{2}\cdot Qq}{2\pi\varepsilon_0 L^2}$$

so dividing this by m gives the initial acceleration of the charge $-q$.

SECTION II, ELECTRICITY AND MAGNETISM

1. (a) Due to the right-hand rule, the magnetic force will force the positive particles to the bottom of the cylinder and the negative particles to the top. This will create an electric field pointing toward the top of the cylinder. As the negative and positive particles accumulate on their respective sides, this will increase the electric field and therefore the electric force. This will continue until the electric force (pushing positive particles to the top and negative ones to the bottom) and the magnetic force (pushing positive particles to the bottom and negative ones to the top) reach an equilibrium.

(b) This situation is similar to a capacitor, so use the equation $V = Ed$. From part (a), we know the magnetic force and electric force will eventually balance, meaning $F_B = F_E$.

$$F_B = F_E \rightarrow qE = qvB \rightarrow E = vB$$

Combining this with $V = Ed$ gives

$$V = (vB)d \rightarrow v = V/(Bd)$$

(c) Magnetic forces can never do work, so the answer is 0 J.

(d) Applying the right-hand rule to each side of the particle yields opposite results. The positive end of the particle will experience a push downward, and the negative end will experience a push upward. This means the net force will be zero, resulting in no translational motion. However, forces applied to an object in this way will result in a non-zero net torque. Thus, the object will begin to rotate clockwise.

2. (a) (i) The current decays exponentially according to the equation

$$I(t) = \frac{\mathcal{E}}{r+R} e^{-t/(r+R)C}$$

where the initial current, $I(0)$, is equal to $\mathcal{E}/(r + R)$. To find the time t at which $I(t) = I(0)/k$, we solve the following equation:

$$\begin{aligned}
\frac{\mathcal{E}}{r+R} e^{-t/(r+R)C} &= \frac{1}{k} \cdot \frac{\mathcal{E}}{r+R} \\
e^{-t/(r+R)C} &= \frac{1}{k} \\
-\frac{t}{(r+R)C} &= \ln \frac{1}{k} \\
-\frac{t}{(r+R)C} &= -\ln k \\
t &= (\ln k)(r+R)C
\end{aligned}$$

(ii) The charge on the capacitor increases according to the equation

$$Q(t) = C\mathcal{E}\left[1 - e^{-t/(r+R)C}\right]$$

The maximum value is $C\mathcal{E}$ (when the capacitor is fully charged). To find the time t at which $Q(t) = C\mathcal{E}/k$, we solve the following equation:

$$\begin{aligned}
C\mathcal{E}\left[1 - e^{-t/(r+R)C}\right] &= \frac{1}{k} C\mathcal{E} \\
1 - e^{-t/(r+R)C} &= \frac{1}{k} \\
e^{-t/(r+R)C} &= 1 - \frac{1}{k} \\
-\frac{t}{(r+R)C} &= \ln \frac{k-1}{k} \\
\frac{t}{(r+R)C} &= \ln \frac{k}{k-1} \\
t &= \left(\ln \frac{k}{k-1}\right)(r+R)C
\end{aligned}$$

(iii) The field energy stored in the capacitor can be written in the form $U_E(t) = [Q(t)]^2/(2C)$, where $Q(t)$ is the function of time given in part (ii) above. We are asked to find the time t at which

$$\begin{aligned}
U(t) &= \frac{1}{k}U_{\text{max}} \\
\frac{[Q(t)]^2}{2C} &= \frac{1}{k}\cdot\frac{(C\mathcal{E})^2}{2C} \\
\left\{C\mathcal{E}\left[1-e^{-t/(r+R)C}\right]\right\}^2 &= \frac{1}{k}(C\mathcal{E})^2 \\
1-e^{-t/(r+R)C} &= \frac{1}{\sqrt{k}} \\
e^{-t/(r+R)C} &= 1-\frac{1}{\sqrt{k}} \\
-\frac{t}{(r+R)C} &= \ln\frac{\sqrt{k}-1}{\sqrt{k}} \\
\frac{t}{(r+R)C} &= \ln\frac{\sqrt{k}}{\sqrt{k}-1} \\
t &= \left(\ln\frac{\sqrt{k}}{\sqrt{k}-1}\right)(r+R)C
\end{aligned}$$

(b) (i) Once the switch is turned to position 2, the capacitor discharges through resistor R(only). Since the capacitor is fully charged (to $V = \mathcal{E}$) when the switch is moved to position 2, the current will decrease exponentially according to the equation

$$I(t) = I_0 e^{-t/RC}$$

where $I_0 = I_{max} = \varepsilon/R$. To find the time at which I is equal to I_0/k, we solve the following equation:

$$\begin{aligned}
I_0 e^{-t/RC} &= \frac{1}{k}I_0 \\
e^{-t/RC} &= \frac{1}{k} \\
-\frac{t}{RC} &= \ln\frac{1}{k} \\
\frac{t}{RC} &= \ln k \\
t &= (\ln k)RC
\end{aligned}$$

(ii) Since the charge on the capacitor obeys the same equation as the current—as given in (b) (i) above, simply replacing I by Q—the time t at which the charge drops to $1/k$ its initial value is the same as the time t at which the current drops to $1/k$ times its initial value: $t = (\ln k)RC$.

3. (a) The magnitude of the average induced emf is given by the equation

$$\begin{aligned}
\mathcal{E}_{\text{avg}} &= \frac{-\Delta\Phi_B}{\Delta t} = \frac{-\left(BA\cos\theta_f - BA\cos\theta_0\right)}{t_f - t_0} \\
&= -\frac{B\left(\pi r^2\right)\cos(90) - B\left(\pi r^2\right)\cos(0)}{T-0} \\
&= \frac{\pi B r^2}{T}
\end{aligned}$$

(b) During the rotation, the upward magnetic flux through the ring would be decreasing, so the induced current would have to be in a direction that produces upward magnetic flux.

Using the right-hand rule, that means the current must run counter-clockwise.

(c) Use Ohm's Law:

$$V = IR \rightarrow \varepsilon_{avg} = I_{avg}R \rightarrow I_{avg} = \varepsilon_{avg}/R = (\pi B r^2/T)/R = \pi B r^2/RT$$

(d) First, because the ring rotates steadily through π/2 radians in T seconds, we can say that the angle θ between **B** and **A** at any given time t will be $\pi/2 * t/T = \pi t/(2T)$. From there,

$$|\varepsilon| = d\Phi_B/dt = d/dt\,(BA\cos\theta) = d/dt\,(BA\cos(\pi t/(2T) = BA\, d/dt\, \cos(\pi t/(2T)) = B(\pi r^2)$$
$$(-\sin(\pi t/(2T))(\pi/(2T)) = \pi^2 r^2 B(-\sin(\pi t/(2T))/(2T)$$

Inserting t_{max} for t and dropping the negative sign (since we want magnitude) then gives

$$\pi^2 r^2 B\sin(\pi t_{max}/(2T))/(2T)$$

(e) The maximum value of a function can be found by taking the derivative and setting it equal to 0. Therefore, using the work from (d) as a launchpad, we can say the equation we want to maximize is

$$|\varepsilon| = \pi^2 r^2 B(-\sin(\pi t/(2T))/(2T)$$

Taking the derivative gives

$$d/dt\, \pi^2 r^2 B(-\sin(\pi t/(2T))/(2T) = \pi^2 r^2 B\, d/dt -\sin(\pi t/(2T))/(2T) = \pi^2 r^2 B\,(-\cos(\pi t/(2T))/(2T)$$
$$(\pi/(2T))$$

Finally, setting this equal to 0 will give an equation that could be solved for the proper time, so the answer is

$$0 = \pi^3 r^2 B(-\cos(\pi t/(2T))/(4T^2) \rightarrow \cos(\pi t/(2T)) = 0 \rightarrow \pi t/(2T) = \pi/2 \rightarrow t = T$$