AP® Calculus AB Exam

SECTION I: Multiple-Choice Questions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Grade
50%
Writing Instrument
Pencil required

Instructions

Section I of this examination contains 45 multiple-choice questions. Fill in only the ovals for numbers 1 through 45 on your answer sheet.

CALCULATORS MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question

Chicago is a

(A) state
(B) city
(C) country
(D) continent
(E) village

Sample Answer

Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.

CALCULUS AB

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Find the second derivative of $x^2y = 2$.

(A) $\frac{6y}{x^2}$

(B) $\frac{x^2}{y}$

(C) $\frac{y}{x^2}$

(D) $-\frac{6y}{x^2}$

(E) $-\frac{x^2}{6y}$

---

2. If $y = \ln(6x^3 - 2x^2)$, then $f'(x) =$

(A) $\frac{9x+2}{3x^2-x}$

(B) $\frac{9x+2}{3x^2+x}$

(C) $\frac{9x-2}{3x^2-x}$

(D) $\frac{9x+2}{3x^2+x}$

(E) $\frac{18x^2+4x}{6x^3-2x^2}$

---

3. Find $\lim\limits_{x\to\infty} 3xe^{-3x}$.

(A) $\frac{1}{3}$

(B) 3

(C) –1

(D) 1

(E) 0

---

4. The radius of a sphere is measured to be 5 cm with an error of ± 0.1 cm. Use differentials to approximate the error in the volume.

(A) $\pm\pi \text{cm}^3$

(B) $\pm 100\pi \text{cm}^3$

(C) $\pm 10\pi \text{cm}^3$

(D) $\pm 4\pi \text{cm}^3$

(E) $\pm 40\pi \text{cm}^3$

---

5. A side of a cube is measured to be 10 cm. Estimate the change in surface area of the cube when the side shrinks to 9.8 cm.

(A) $+2.4\ \text{cm}^2$

(B) $-2.4\ \text{cm}^2$

(C) $-120\ \text{cm}^2$

(D) $+24\ \text{cm}^2$

(E) $-24\ \text{cm}^2$

---

6. Find the derivative of y, when $y^2 = (x^2 + 2)(x + 3)^2(2x + 7)^{\frac{1}{2}}$ at (1,12)?

(A) $\frac{20}{3}$

(B) 7

(C) $\frac{22}{3}$

(D) $\frac{23}{3}$

(E) 8

---

7. $\int \frac{x^3}{2} dx =$

(A) $\frac{x^4}{8} + C$

(B) $\frac{x^4}{2} + C$

(C) $2x^4 + C$

(D) $\frac{3}{2}x^2 + C$

(E) $8x^4 + C$

---

8. $\int x^2 \sin(3x^3 + 2)dx =$

(A) $-9\cos(3x^3 + 2) + C$

(B) $-\cos(3x^3 + 2) + C$

(C) $\frac{-\cos\left(3x^3+2\right)}{9} + C$

(D) $\dfrac{\cos\left(3x^3+2\right)}{9}$ + C

(E) 9cos($3x^3$ + 2) + C

---

9. If $f(x)$ = $\begin{cases} 2ax^2+bx+6, & x\le -1 \\ 3ax^3-2bx^2+4x, & x>-1 \end{cases}$ and is differentiable for all real values, then b = ?

(A) –13

(B) 0

(C) 45

(D) 55

(E) 110

---

10. $\dfrac{d}{dx}\left(\dfrac{x^3-4x^2+3x}{x^2+4x-21}\right)=$

(A) $\dfrac{x^2-x}{x+7}$

(B) $\dfrac{x-1}{x-7}$

(C) $\dfrac{x^2-14x+7}{(x-7)^2}$

(D) $\dfrac{2x^2+13x-7}{(x+7)^2}$

(E) $\dfrac{x^2+14x-7}{(x+7)^2}$

---

11. $\lim\limits_{h\to 0}\dfrac{2x^2+4xh+2h^2-2x^2}{h}$

(A) 4

(B) $3x^2$

(C) $2x^2$

(D) $4x$

(E) $6x$

---

12. Find the point on the curve x^2 + y^2 = 9 that is a minimum distance from the point (1,2).

(A) $(\sqrt{5},\ 2)$

(B) $(-\sqrt{5},\ -2)$

(C) $(\sqrt{5},\ -2)$

(D) $(-\sqrt{5},\ 2)$

(E) (5, 2)

---

13. Find $\dfrac{dy}{dx}$ if y = $\log_3(2x^3 + 4x^2)$

(A) $\dfrac{6x^2+8x}{\left(x^2+2x\right)\ln 3}$

(B) $\frac{3x+4}{\left(2x^3+4x^2\right)\ln 3}$

(C) $\frac{3x+4}{\left(x^2+2x\right)\ln 3}$

(D) $\frac{3x+4}{3\ln\left(x^2+2x\right)}$

(E) $\frac{6x^2+8x}{\left(3x^3+2x^2\right)\ln 3}$

---

14. What curve is represented by $x = 2t^3$ and $y = 4t^9$?

(A) $y = 2x^2$

(B) $y = x^2$

(C) $y = 3x^2$

(D) $y = x^3$

(E) $y = 2x^3$

---

15. Find $\lim_{x\to 0}\frac{2x^3-3\sin x}{x^4}$.

(A) -1

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

(E) 1

---

16. $\int 18x^2\sec^2(3x^3)dx =$

(A) $2\tan^2(3x^3) + C$

(B) $2\cot^2(3x^3) + C$

(C) $\cot(3x^3) + C$

(D) $\tan(3x^3) + C$

(E) $2\tan(3x^3) + C$

---

17. What is the equation of the line normal to the curve $y = x^3 + 2x^2 - 5x + 7$ at $x = 1$?

(A) $y = -\frac{1}{2}x + \frac{11}{2}$

(B) $y = 2x + 3$

(C) $y = -\frac{1}{2}x - \frac{11}{2}$

(D) $y = -2x + 3$

(E) $y = -2x - \frac{11}{2}$

---

18. Find the value of c that satisfies Rolle's Theorem for $f(x) = \dfrac{x^2+4x-12}{x^2+2x-3}$ on the interval $[-6,2]$.

(A) -6

(B) -3

(C) 1

(D) 2

(E) No such value exists.

---

19. If $\cos^2 x + \sin^2 y = y$, then $\dfrac{dy}{dx}$.

(A) $\dfrac{2\cos x \sin x}{2\cos y \sin y + 1}$

(B) $\dfrac{\cos x \sin x}{\cos y \sin y}$

(C) $\dfrac{2\cos x \sin x}{2\cos y \sin y - 1}$

(D) $\dfrac{\sin y \cos y}{1 - \cos x \sin x}$

(E) $\dfrac{2\cos y \sin y}{2\cos x \sin x - 1}$

---

20. If $f(x) = e^{3x}$, then $f''(\ln 3) =$

(A) 9

(B) 27

(C) 81

(D) 243

(E) 729

---

21. Find $\dfrac{dy}{dx}$ if $2y^2 - 6y = x^4 + 2x^3 - 2x - 5$ at $(1,1)$.

(A) -1

(B) -2

(C) -3

(D) -4

(E) -5

---

22. Find $\dfrac{dy}{dx}$ if $2\sin^3 y + 2\cos^3 x = 2\cos^3 y - 4\sin^3 x$.

(A) $-\dfrac{\sin 2x(\cos x + 2\sin x)}{\sin 2y(\sin y + \cos y)}$

(B) $\dfrac{\sin 2x(\cos x + 2\sin x)}{\sin 2y(\sin y + \cos y)}$

(C) $\dfrac{\sin x \cos x(\cos x + 2\sin x)}{\sin y \cos y(\sin y + \cos y)}$

(D) $-\dfrac{\sin 2x}{\sin 2y}$

(E) $\dfrac{\cos 2x(\cos x + 2\sin x)}{\cos 2y(\sin y + \cos y)}$

23. $\int \frac{\ln^3 x}{x} dx =$

(A) $\frac{\ln^3 x}{3} + C$

(B) $\frac{\ln^4 x}{4} + C$

(C) $\frac{\ln^5 x}{5} + C$

(D) $\ln^3 x + C$

(E) $\ln^4 x + C$

---

24. Find the volume of the region formed by the curve $y = x^2$ the x-axis, and the line $x = 3$ when revolved around the y-axis.

(A) $\frac{3}{2}\pi$

(B) $\frac{9}{2}\pi$

(C) $\frac{27}{2}\pi$

(D) $\frac{81}{2}\pi$

(E) $\frac{243}{2}\pi$

---

25. $\int_0^4 x^3 dx =$

(A) 16

(B) 32

(C) 48

(D) 56

(E) 64

---

26. Is the function $f(x) = \begin{cases} x^3 - 3, & x < 3 \\ 2x + 7, & x \geq 3 \end{cases}$ continuous at $x = 3$? If not, what is the discontinuity?

(A) The function is continuous.

(B) Point

(C) Essential

(D) Jump

(E) Removable

---

27. Where does the curve $y = 5 - (x - 2)^{\frac{2}{3}}$ have a cusp?

(A) (0,5)

(B) (5,2)

(C) (2,5)

(D) (5,0)

(E) There is no cusp.

28. $\int (x^2 + 2x)\cos(x^3 + 3x^2)\,dx =$

(A) $\sin(3x^2 + 6x) + C$

(B) $-\frac{1}{3}\sin(x^3 + 3x^2) + C$

(C) $-\sin(x^3 + 3x^2) + C$

(D) $\sin(x^3 + 3x^2) + C$

(E) $\frac{1}{3}\sin(x^3 + 3x^2) + C$

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

---

29. An open top cylinder has a volume of $125\pi \text{in}^3$. Find the radius required to minimize the amount of material to make the cylinder.

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

---

30. If the position of a particle is given by $x(t) = 2t^3 - 5t^2 + 4t + 6$, where $t > 0$. What is the distance traveled by the particle from $t = 0$ to $t = 3$?

(A) $\frac{1}{27}$

(B) $\frac{28}{27}$

(C) 20

(D) 21

(E) $\frac{569}{27}$

---

31. At what times, t, are the x- and y-components of the particle's velocity equal if the curve is represented by $x = 2t^3 + 3t^2 - 5$ and $y = t^4 - 4t^3 + 7t^2$?

(A) $t = 0$

(B) $t = \frac{1}{2}$

(C) $t = 4$

(D) $t = 0$ and $t = \frac{1}{2}$

(E) $t = 0$, $t = \frac{1}{2}$, and $t = 4$

---

32. Find the equation of the line tangent to the graph of $y = 2x - 3x^{-\frac{2}{3}} + 5$ at $x = 8$.

(A) $y = \frac{33}{16}x + \frac{15}{4}$

(B) $y = \frac{15}{4}x + \frac{33}{16}$

(C) $y = \frac{16}{33}x + \frac{4}{15}$

(D) $y = \frac{16}{33}x + \frac{15}{4}$

(E) $y = \frac{33}{16}x + \frac{4}{15}$

---

33. Which point on the curve $y = 5x^3 - 12x^2 - 12x + 64$ has a tangent that is parallel to $y = 3$?

(A) $(0, -2)$

(B) $(2, 32)$

(C) $(\frac{2}{5}, 12)$

(D) $(-2, \frac{288}{25})$

(E) $\left(\frac{2}{5}, \frac{256}{25}\right)$

---

34. A 50 foot ladder is leaning against a building and being pulled to the ground, so the top is sliding down the building. If the rate the bottom of the ladder is being pulled across the ground is 12 ft/sec, what is the rate of the top of the ladder sliding down the building when the top is 30 ft from the ground?

(A) 12 ft/sec

(B) 9 ft/sec

(C) 20 ft/sec

(D) 9.6 ft/sec

(E) 16 ft/sec

---

35. What is the distance traveled from $t = 0$ to $t = 4$ given the position function, $x(t) = 2t^3 - 9t^2 + 12t + 13$?

(A) 30 units

(B) 32 units

(C) 33 units

(D) 34 units

(E) 35 units

---

36. The tangent to a curve described by $x = 3t^3 - 5t + 2$ and $y = 7t^2 - 16$ is what at $t = 1$?

(A) $-7x + 2y = -18$

(B) $2x - 7y = 18$

(C) $7x + 2y = 18$

(D) $2x + 7y = -18$

(E) $7x - 2y = -18$

---

37. Approximate $\sqrt{16.04}$.

(A) 4.005

(B) 4.04

(C) 4.02

(D) 4.002

(E) 4.05

---

38. Approximate the area under the curve $y = x^2 + 2$ from $x = 1$ to $x = 2$ using four midpoint rectangles.

(A) 4.333

(B) 3.969

(C) 4.719

(D) 4.344

(E) 4.328

---

39. Find the area under the curve $y = x^2 + 2$ from $x = 1$ to $x = 2$.

(A) 4.333

(B) 3.969

(C) 4.719

(D) 4.344

(E) 4.328

---

40. $\int \frac{3x-2}{(x+2)^2}dx =$

(A) $\ln|x + 2| + \frac{1}{x+2} + C$

(B) $3\ln|x + 2| + \frac{4}{x+2} + C$

(C) $3\ln|x + 2| - \frac{4}{x+2} + C$

(D) $-3\ln|x + 2| - \frac{4}{x+2} + C$

(E) $-3\ln|x + 2| + \frac{4}{x+2} + C$

---

41. The side of a cube is increasing at a rate of 3 inches per second. At the instant when the side of the cube is 6 inches long. What is the rate of change (in inches/second) of the surface area of the cube?

(A) 108

(B) 216

(C) 324

(D) 648

(E) 1296

---

42. If the position of a particle is given by $x(t) = 3t^3 - 2t^2 - 16$ where $t > 0$. When does the particle change direction?

(A) $\frac{2}{3}$

(B) $\frac{4}{3}$

(C) $\frac{9}{4}$

(D) 2

(E) 3

---

43. The radius of a sphere is increased from 9 cm to 9.05 cm. Estimate the change in volume.

(A) 1.25×10^{-4} cm^3

(B) 11.3097 cm^3

(C) 16.965 cm^3

(D) 50.894 cm^3

(E) 152.681 cm^2

---

44. Find an equation of the line tangent to the curve represented by $x = 4\cos t + 2$ and $y = 2\sin t$ and $t = \frac{\pi}{3}$.

(A) $y = \frac{\sqrt{3}}{6}x + \frac{5\sqrt{3}}{3}$

(B) $y = -\frac{\sqrt{3}}{6}x + \frac{2\sqrt{3}}{3}$

(C) $y = \frac{\sqrt{3}}{6}x + \sqrt{3}$

(D) $y = -\frac{\sqrt{3}}{6}x + \frac{5\sqrt{3}}{3}$

(E) $y = -\frac{\sqrt{3}}{6}x - \frac{\sqrt{3}}{3}$

---

45. Use differentials to approximate $\sqrt{4.002}$.

(A) 2

(B) 2.0005

(C) 2.005

(D) 2.05

(E) 2.5

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II

GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt (X2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II, PART A

Time—30 minutes

Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

---

1. Water is dripping from a pipe into a container whose volume increases at a rate of The water takes the shape of a cone with both its radius and height changing with time.

   (a) What is the rate of change of the radius of the water at the instant the height is 2 cm and the radius is 5 cm? At this instant the height is changing at a rate of 0.5 cm/min.

   (b) The water begins to be extracted from the container at a rate of $E(t) = 75t^{0.25}$. Water continues to drip from the pipe at the same rate as before. When is the water at its maximum volume? Justify your reasoning.

   (c) By the time water began to be extracted, 3000 cm^3 of water had already leaked from the pipe. Write, but do not evaluate, an expression with an integral that gives the volume of water in the container at the time in part (b).

---

2. The temperature in a room increases at a rate of $\dfrac{dT}{dt} = kT$, where k is a constant.

   (a) Find an equation for T, the temperature (in °F), in terms of t, the number of hours passed, if the temperature is 65 °F initially and 70 °F after one hour.

   (b) How many hours will it take for the temperature to reach 85 °F?

   (c) After the temperature reaches 85 °F, a fan is turned on and cools the room at a consistent rate of 7 °F/hour. How long will it take for the room to reach 0 °F?

SECTION II, PART B

Time—1 hour

Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

---

3. Let R be the region enclosed by the graphs of $y = \frac{2}{x+1}$, $y = x^2$, and the lines $x = 0$ and $x = 1$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) Set up, but do not evaluate, the expression for the volume of the solid generated when R is revolved around the line $x = 2$.

---

4. Consider the equation $x^3 + 2x^2y + 4y^2 = 12$

   (a) Write an equation for the slope of the curve at any point (x,y).

   (b) Find the equation of the tangent line to the curve at $x = 0$.

   (c) If the equation given for the curve is the path a car travels in feet over t seconds, find $\frac{d^2y}{dx^2}$ at $(0, \sqrt{3})$ and explain what it represents with proper units.

---

5. Water is filling at a rate of 64π in^3 into a conical tank that has a diameter of 36 in at its base and whose height is 60 in.

   (a) Find an expression for the volume of water (in in^3) in the tank in terms of its radius.

   (b) At what rate is the radius of the water expanding when the radius is 20 in.

   (c) How fast in (in/sec) is the height of the water increasing in the tank when the radius is 20 in?

---

6. If a ball is accelerating at a rate given by $a(t) = -64\frac{\text{ft}}{\text{sec}^2}$, the velocity of the ball is $96\frac{\text{ft}}{\text{sec}}$ at time $t = 1$, and the height of the ball is 100 ft at $t = 0$, what is

   (a) The equation of the ball's velocity at time t?

   (b) The time when the ball is changing direction?

   (c) The equation of the ball's height?

   (d) The ball's maximum height?

STOP

END OF EXAM

ANSWER KEY

Section I

1. A
2. C
3. E
4. C
5. E
6. C
7. A
8. C
9. D
10. E
11. D
12. A
13. C
14. B
15. C
16. E
17. A
18. E
19. C
20. D
21. D
22. A
23. B
24. D
25. E
26. D
27. C
28. E
29. D
30. E
31. E
32. A
33. B
34. E
35. D
36. A
37. A
38. E
39. A
40. B
41. B
42. A
43. D
44. D
45. B

EXPLANATIONS

Section I

1. **A** First, use implicit differentiation to find $\frac{dy}{dx}$:

$$x^2 \frac{dy}{dx} + 2xy = 0$$

Isolate $\frac{dy}{dx}$ and simplify:

$$\frac{dy}{dx} = \frac{-2xy}{x^2} = \frac{-2y}{x}$$

Next, take the second derivative via implicit differentiation:

$$\frac{d^2y}{dx^2} = \frac{x\left(-2\frac{dy}{dx}\right) - (-2y)(1)}{x^2} = \frac{-2x\frac{dy}{dx} + 2y}{x^2}$$

Plug in $\frac{dy}{dx}$ and simplify:

$$\frac{d^2y}{dx^2} = \frac{-2x\left(\frac{-2y}{x}\right) + 2y}{x^2} = \frac{4y + 2y}{x^2} = \frac{6y}{x^2}$$

2. **C** When $y = \ln u$, $\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx}$. For this problem, $u = 6x^3 - 2x^2$ and $\frac{du}{dx} = 18x^2 - 4x$. Then, $\frac{dy}{dx} = \frac{18x^2 - 4x}{6x^3 - 2x^2} = \frac{9x - 2}{3x^2 - x}$.

3. **E** When you insert ∞ for x, the limit is $\frac{\infty}{\infty}$, which is indeterminate. First, rewrite the limit as $\lim_{x\to\infty} \frac{3x}{e^{3x}}$. Then, use L'Hôpital's Rule to evaluate the limit: $\lim_{x\to 0} \frac{3}{3e^{3x}}$. This limit exists and equals 0.

4. **C** In order to approximate the error in the volume of the sphere use the approximation formula: $dy = f'(x)dx$. For this problem, $f(x) = V = \frac{4}{3}\pi r^3$, $f'(x) = \frac{dV}{dr} = 4\pi r^2 = 100\pi$, and $dx = dr = \pm 0.1$. When these values are input, the equation is $dy = dV = 100\pi(\pm 0.1) = \pm 10\pi \text{ cm}^3$.

5. **E** In order to approximate the change in the surface area of the cube, use the approximation formula: $dy = f'(x)dx$. For this problem, $f(x) = A = 6s^2$, $f'(x) = \frac{dA}{ds} = 12s = 120$, and $dx = ds = -0.2$. When these values are input, the equation is $dy = dA = 120(-0.2) = -24 \text{ cm}^2$.

6. **C** This problem can be solved with implicit differentiation, but that can get especially messy. We are going to use logarithmic differentiation to solve it. First, take the natural log of both sides: $\ln y^2 = \ln\left((x^2+2)(x+3)^2(2x+7)^{\frac{1}{2}}\right)$. Use logarithmic rules to simplify the equation: $2 \ln y = \ln(x^2 + 2) + 2 \ln(x + 3) + \frac{1}{2}\ln(2x + 7)$. Now, differentiate both sides with respect to x: $\frac{2}{y}\frac{dy}{dx} = \frac{2x}{x^2+2} + \frac{2}{x+3} + \frac{1}{4x+14}$.

Next, isolate $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{y}{2}\left(\frac{2x}{x^2+2} + \frac{2}{x+3} + \frac{1}{4x+14}\right)$. Finally, plug in the given values for x and y and solve, so $\frac{dy}{dx} = 88$.

7. **A** Use the Power rule to integrate: $\int \frac{x^3}{2} dx = \frac{1}{4}\left(\frac{x^4}{2}\right) = \frac{x^4}{8} + C$.

8. **C** Use u-substitution. Here, and $u = 3x^3 + 2$ and $du = 9x^2\, dx$. Then, $\int x^2 \sin(3x^3 + 2)\, dx = \frac{1}{9}\int \sin x\, dx = \frac{1}{9}(-\cos u) + C = \frac{-\cos u}{9} + C$. Replace u for the final solution: $\_\frac{\cos\left(3x^3+2\right)}{8} + C$

9. **D** First take the derivative of $f(x)$. $f'(x) = \begin{cases} 4ax + b, & x \le -1 \\ 9ax^2 - 4bx + 4, & x > -1 \end{cases}$. In order for $f(x)$ to be differentiable for all real values, both pieces of $f(x)$ must be equal at $x = -1$ and both pieces of $f'(x)$ must be equal at $x = -1$. Therefore, plug $x = -1$ into both $f(x)$ and $f'(x)$. $f(-1) = \begin{cases} 2a - b + 6 \\ -3a - 2b - 4 \end{cases}$ and $f'(-1) = \begin{cases} -4a + b \\ 9a + 4b + 4 \end{cases}$. When the two parts of $f(-1)$ are set equal to each other, $10 = -5a - b$ and when the two parts of $f'(-1)$ are set equal to each other, $-4 = 13a + 3b$. When this system is solved, $a = -13$ and $b = 55$.

10. **E**
$$\frac{d}{dx}\left(\frac{x^3 - 4x^2 + 3x}{x^2 + 4x - 21}\right) = \frac{d}{dx}\left(\frac{x(x-3)(x-1)}{(x-3)(x+7)}\right)$$
$$= \frac{d}{dx}\left(\frac{x(x-1)}{(x+7)}\right) = \frac{d}{dx}\left(\frac{x^2 - x}{(x+7)}\right)$$
$$= \left(\frac{(2x-1)(x+7)(x^2-x)}{(x+7)^2}\right)$$

11. **D** Notice the limit is in the form of the definition of the derivative. You could evaluate the limit, but if you see the definition of the derivative and the main function, $f(x) = 2x^2$, it is easier to evaluate the derivative directly. Thus, the solution is $f'(x) = 4x$.

12. **A** Use the distance formula ($D^2 = (x - x_1)^2 + (y - y_1)^2$) to determine the distance between the curve and the point: $D^2 = (x - 1)^2 + (y - 2)^2 = x^2 - 2x + 1 + y^2 - 4y + 4$. From the equation of the curve, you can solve for y^2 and y: $y^2 = 9 - x^2$ and $y = \sqrt{\left(9 - x^2\right)}$. Substitute these back into the formula for distance:

$$D^2 = x^2 - 2x + 1 + 9 - x^2 - 4\sqrt{\left(9 - x^2\right)} + 4.$$

To simplify calculations, recall that the functions and D^2 will be minimized at the same point, so instead of solving for D, continue calculations with D^2 which will now be called L: $D^2 = L$. Thus, $L = 14 - 2x - 4\sqrt{9 - x^2}$. To minimize the distance, take the first derivative of L and set it equal to zero: $\frac{dL}{dx} = -2 + \frac{4}{\sqrt{9 - x^2}} = 0$. Solve this equation for x: $x = \pm\sqrt{5}$. Find the corresponding y-values: $y = \pm 2$. The point closest to (1,2) would then be $(\sqrt{5}, 2)$. You can verify this with the second derivative test.

13. **C** If $y = \log_a u$, then $\frac{dy}{dx} = \frac{1}{u \ln a}\frac{du}{dx}$. In this case, $\frac{dy}{dx} = \frac{6x^2 + 8x}{\left(2x^3 + 4x^2\right)\ln 3} = \frac{3x + 4}{\left(x^2 + 2x\right)\ln 3}$.

14. **B** When dealing with parametric functions, your task is to eliminate t. In this case, notice $y = 4t^9 = (2t^3)^2$. Since $x = 2t^3$, $y = x^2$.

15. **C** Use l’Hôpital’s Rule.

$$\lim_{x\to 0}\frac{2x^3-3\sin x}{x^4}=\lim_{x\to 0}\frac{6x^2-3\cos x}{4x^3}=\lim_{x\to 0}\frac{12x+3\sin x}{12x^2}$$

$$=\lim_{x\to 0}\frac{12x+3\cos x}{24x}=\lim_{x\to 0}\frac{-3\sin x}{24}=0$$

16. **E** Use u-substitution where $u = 3x^3$ and $du = 9x^2$. Therefore,

$$\int 18x^2\sec^2(3x^3)dx = 2\int \sec^2 u\,du = 2\tan u + C = 2\tan(3x^3) + C.$$

17. A. First, plug $x = 1$ into the equation and solve for y; $y = 5$. Next, take the first derivative of y: $\frac{dy}{dx} = 3x^2 + 4x - 5$. Solve for $\frac{dy}{dx}$ at $x = 1$: $\frac{dy}{dx} = 2$. The slope of the normal line is $-\frac{1}{2}$. Thus, the equation of the normal line is $y - 5 = -\frac{1}{2}(x - 1)$. The equation simplified is $y = -\frac{1}{2}x+\frac{11}{2}$.

18. **E** Factor $f(x)$ and notice there are discontinuities over the interval: $f(x) = \frac{(x-2)(x+6)}{(x-1)(x+3)}$. The discontinuities are located at $x = 1$ and $x = -3$. Because there are discontinuities over the integral, Rolle's Theorem cannot be applied.

19. **C** Using implicit differentiation, you can evaluate this equation: $-2\cos x\sin x + 2\sin y\cos y\frac{dy}{dx} = \frac{dy}{dx}$. After simplifying, $\frac{dy}{dx}=\frac{2\cos x\sin x}{2\cos y\sin y-1}$.

20. **D** Via the chain rule, $f'(x) = 3e^{3x}$ and $f''(x) = 9e^{3x}$. Plugging in ln 3 for x, results in $f''(\ln 3) = 9e^{3(\ln 3)} = 9e^{\ln 3^3} = 9e^{\ln 27} = 9 \cdot 27 = 243$.

21. **D** Use implicit differentiation and plug in for the point (1,1):

$$4y\frac{dy}{dx} - 6\frac{dy}{dx} = 4x^3 + 6x^2 - 2$$

$$4(1)\frac{dy}{dx} - 6\frac{dy}{dx} = 4(1)^3 + 6(1)^2 - 2$$

$$-2\frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = -4$$

22. **A** Use implicit differentiation: $6\sin^2 y\cos y\frac{dy}{dx} - 6\sin x\cos^2 x = -6\sin y\cos^2 y\frac{dy}{dx} - 12\sin^2 x\cos x$. In addition, recall that $\sin 2x = 2\sin x\cos x$. When the equation is rearranged and that trig identity used, $\frac{dy}{dx}=-\frac{\sin 2x(\cos x+2\sin x)}{\sin 2y(\sin y+\cos y)}$

23. **B** Using u-substitution, $\int\frac{\ln^3 x}{x}dx=\frac{\ln^4 x}{4}+C$.

24. **D** Because the region is bound by three curves given in the form $y =$ and $x =$, it is likely better to use the cylindrical shells method to solve this problem: $2\pi\int_0^3 x(x^2-0)dx = 2\pi\int_0^3 x^3dx=\frac{81\pi}{2}$.

25. **E** Follow the First Fundamental Theorem of Calculus: $\int_0^4 x^3dx=\left.\frac{x^4}{4}\right|_0^4=\frac{256}{4}=64$.

26. **D** There are three conditions that must be satisfied for a function to be continuous: 1. $f(c)$ exists. 2. $\lim_{x \to c} f(x)$ exists. 3. $\lim_{x \to c} f(x) = f(c)$. For this function, condition 1 is met as $f(3) = 13$. Condition 2, however, is violated; from the left the limit equals 24 and from the right it equals 13. This signifies a jump discontinuity.

27. **C** If the derivative of a function approaches infinity and negative infinity from both sides of a point and it is continuous at that point, then there is a cusp. The graph of $y = 5 - (x - 2)^{\frac{2}{3}}$ is continuous. Next, check the points where the derivative is undefined or zero. In this case, $\frac{dy}{dx} = -\frac{2}{3}(x-2)^{-\frac{1}{3}}$ is undefined at $x = 2$, and zero nowhere. To determine whether there is a cusp at $x = 2$, we need to check the limit of $\frac{dy}{dx}$ as x approaches 2 from the left and right. $\lim_{x \to 2+} -\frac{2}{3}(x-2)^{-\frac{1}{3}} = -\infty$ and $\lim_{x \to 2-} -\frac{2}{3}(x-2)^{-\frac{1}{3}} = \infty$. Thus, there is a cusp at $x = 2$, when you plug in 2 to the equation for y, the location of the cusp is (2,5).

28. **E** Solve the integral using u-substitution: $u = x^3 + 3x^2$ and $du = (3x^2 + 6x)\,dx$.

$$\int (x^2 + 2x)\cos(x^3 + 3x^2)\,dx = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin(x^3 + 3x^2) + C.$$

29. **D** The amount of material required to make this cylinder corresponds with the surface area of the cylinder found by $S = \pi r^2 + 2\pi rh$. As the problem gave the volume of the cylinder and only asked for the radius, use the volume to eliminate h. $V = \pi r^2 h = 125\pi$. Thus, $h = \frac{125}{r^2}$. Plug this expression for h into the equation for the surface area: $S = \pi r^2 + \frac{250\pi}{r}$. Next, to minimize the amount of material, take the first derivative of S and set it equal to zero to determine the critical points for r: $\frac{dS}{dr} = 2\pi r - \frac{250\pi}{r^2} = 0$. Thus, the critical point is at $r = 5$. To verify that this value of r minimizes the material, take the second derivative and ensure that the second derivative is positive at $r = 5$. $\frac{d^2S}{dr^2} = 2\pi + \frac{500\pi}{r^3}$. At $r = 5$, $\frac{d^2S}{dr^2} = 6\pi$, so $r = 5$ minimizes the amount of material.

30. **E** To determine the distance traveled by the particle, we need to know the position of the particle at those two times. However, we first need to know whether the particle changes direction at any time over the interval. In other words, we need to know if the velocity is zero over the interval at all. Since the velocity is the first derivative of the position function, we take the first derivative and set it equal to zero: $x'(t) = 6t^2 - 10t + 4 = 0$. Solving for t, the particle changes direction at $t = \frac{2}{3}$ and $t = 1$. Now, the positions at the four times are found: $x(0) = 6$, $x\left(\frac{2}{3}\right) = \frac{190}{27}$, $x(1) = 7$, and $x(3) = 27$. To determine the distance traveled, take the absolute value of the distance traveled over the smaller time intervals and add them together.

$$\left|x\left(\frac{2}{3}\right) - x(0)\right| + \left|x(1) - x\left(\frac{2}{3}\right)\right| + \left|x(3) - x(1)\right| = \frac{28}{27} + \frac{1}{27} + 20 = \frac{569}{27}.$$

31. **E** Take the derivative of the x- and y-components of the position functions with respect to t: $\frac{dx}{dt} = 6t^2 + 6t$ and $\frac{dy}{dt} = 4t^3 - 12t^2 + 14t$. Set those two derivatives equal to each other and solve for t: $6t^2 + 6t = 4t^3 - 12t^2 + 14t$ or $0 = 4t^3 - 18t^2 + 8t$. Solving for t, $t = 0$, $t = \frac{1}{2}$, and $t = 4$.

32. **A** First, calculate the derivative of y at $x = 8$: $\frac{dy}{dx} = 2 + 2x^{-\frac{5}{3}} = 2 + 2(8)^{-\frac{5}{3}} = \frac{33}{16}$. Then, determine y at $x = 8$: $y = 2x - 3x^{-\frac{2}{3}} + 5 = 2(8) - 3(8)^{-\frac{2}{3}} + 5 = \frac{81}{4}$. Finally, plug these values into the point-slope formula: $y - \frac{81}{4} = \frac{33}{16}(x - 8)$, thus, $y = \frac{33}{16}x + \frac{15}{4}$.

33. **B** Take the derivative of $y = 5x^3 - 12x^2 - 12x + 64$ and set it equal to 0, because the slope of $y = 3$ is 0. Thus, $0 = 15x^2 - 24x - 12 = (3x - 6)(5x + 2)$ and $x = 2$ or $-\frac{2}{5}$. Plug these values into the equation and solve for y. The two possible points are then (2,32) and $\left(-\frac{2}{5}, \frac{288}{25}\right)$.

34. **E** The ladder makes a right triangle with the building and the ground, so the relationship between the three can be found using the Pythagorean theorem, in which we will call x the distance the bottom of the ladder is from the building across the ground and y the distance the top of the ladder is from the ground up the building, so $x^2 + y^2 = 50^2$. Since we want to find the rate that the top of the ladder is sliding, we need to differentiate this equation with respect to t: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. We already know $\frac{dx}{dt} = 12$ ft/sec and our y at the time of interest is 30 ft. In order to determine x at that time, plug 30 into $x^2 + y^2 = 50^2$ and solve for x. Thus, $x = 40$ feet. Plug these values into the differentiated equation and solve for $\frac{dy}{dt}$: $2(40)(12) + 2(30)\frac{dy}{dt} = 0$, so $\frac{dy}{dt} = -16$ ft/sec.

35. **D** When determining the distance traveled, first determine whether the velocity changes sign over the specified time interval. If it does, then the distance traveled will need to be found piecewise. Thus, to begin, differentiate $x(t)$ with respect to time to get $v(t)$: $v(t) = 6t^2 - 18t + 12$. Set $v(t)$ equal to zero and determine when the velocity is zero. In this case, the velocity is zero at $t = 1$ and $t = 2$. To confirm that the particle is changing directions at those times, differentiate the velocity with respect to time and determine whether the acceleration is zero at $t = 1$ and $t = 2$. $a(t) = 12t - 18$, $a(1) = -6$, and $a(2) = 6$. Since the acceleration is not zero at either of those times, the particle is changing directions. Therefore, the distance traveled must be found by adding the distance traveled from $t = 0$ to $t = 1$, $t = 1$ to $t = 2$, and $t = 2$ to $t = 4$. The equation should look like this: $|x(1) - x(0)| + |x(2) - x(1)| + |x(4) - x(2)| =$ total distance. Absolute values are used so the directions will not affect the final result, so the total distance is 34 units.

36. **A** The slope of the tangent is $\frac{dy}{dx}$ which is represented parametrically by $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. $\frac{dy}{dt} = 14t$ and $\frac{dx}{dt} = 9t^2 - 5$, so $\frac{dy}{dx} = \frac{14t}{9t^2 - 5}$. At $t = 1$ the slope is $\frac{7}{2}$. At $t = 1$, $x = 0$ and $y = -9$. Therefore, the equation of the tangent to the curve is $y + 9 = \frac{7}{2}(x - 0)$ or $-7x + 2y = -18$.

37. **A** Use a differential to approximate $\sqrt{16.04}$. Recall the general formula is $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$. For this problem, $f(x) = \sqrt{16} = 4$, $f'(x) = \frac{1}{2}(16)^{-\frac{1}{2}} = \frac{1}{8}$, and $\Delta x = 0.04$. When these values are input into the equation is $\sqrt{16.04} \approx 4 + \frac{0.04}{8} \approx 4.005$.

38. **E** The formula for the area under a curve using midpoint rectangles is: $A = \left(\frac{b-a}{n}\right)\left(y_{\frac{1}{2}} + y_{\frac{3}{2}} + y_{\frac{5}{2}} + \ldots + y_{\frac{2n-1}{2}}\right)$, where a and b are the x-values that bound the area and n is the number of rectangles. Since we are interested in the midpoints, the x-coordinates are $x_{\frac{1}{2}} = \frac{9}{8}$, $x_{\frac{3}{2}} = \frac{11}{8}$, $x_{\frac{5}{2}} = \frac{13}{8}$, and $x_{\frac{7}{2}} = \frac{15}{8}$. The y-coordinates are found by plugging these values into the equation for y, so $y_{\frac{1}{2}} = 3.26563$, $y_{\frac{3}{2}} = 3.89063$, $y_{\frac{7}{2}} = 4.64063$, and $y_{\frac{9}{2}} = 5.51563$. Then, $A = \left(\frac{2-1}{4}\right)(3.26563 + 3.89063 + 4.64063 + 5.51563) = 4.32813$.

39. **A** Use the Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$. For this problem, $\int_1^2 (x^2 + 2)dx = \frac{x^3}{3} + 2x\Big|_1^2$.

40. **B** Use the Method of Partial Fractions to evaluate: $\frac{A}{x+2}+\frac{B}{(x+2)^2}=\frac{3x+2}{(x+2)^2}$. Then, $A = 3$ and $B = -4$.

$$\int\frac{3x-2}{(x+2)^2}dx = \int\frac{3}{x+2}dx-\int\frac{4}{(x+2)^2}dx = 3\ln|x+2|+\frac{4}{x+2}+C.$$

41. **B** This is a related rates problem. The surface area of the cube is given by $A = 6s^2$. If you differentiate with respect to time, the function becomes $\frac{dA}{dt}=12s\frac{ds}{dt}$. We are given $s = 6$ and $\frac{ds}{dt} = 3$. We must solve for $\frac{dA}{dt}$. When everything is plugged into the equation, $\frac{dA}{dt} = 216$.

42. **A** Recall, a particle changes direction when its velocity equals zero but its acceleration does not, and recall that and $v(t) = x'(t)$ and $a(t) = x''(t)$. First, take the derivative of $x(t)$, set it equal to zero, and solve for t: $x'(t) = 9t^2 - 4 = 0$ and $t = \frac{2}{3}$. To determine whether the particle changes at that time, take the second derivative and determine the value of the second derivative at $t = \frac{2}{3}$: $x(t) = 18t$ and $x\left(\frac{2}{3}\right) = 12$. Since, $x''(t)$ is not zero when $x'(t)$ is, the particle is changing direction at $t = \frac{2}{3}$.

43. **D** Use this formula for differentials: $dy = f'(x)dx$. The formula for the volume of a sphere is $f(x) = \frac{4}{3}\pi r^3$ and $f'(x) = 4\pi r^2$. In order to estimate the change in volume, we must evaluate the function at $x = 9$ and $dx = 0.05$. When the equation is evaluated using the given equations and values, $dy = 50.894$ cm^3.

44. **D** $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2\cos t}{-4\sin t}=-\frac{\cos t}{2\sin t}$. At $t=\frac{\pi}{3}$, $\frac{dy}{dx}=-\frac{\sqrt{3}}{6}$. At this time, $x = 4$ and $y = \sqrt{3}$. Thus, equation for the tangent line is $(y - \sqrt{3}) = -\frac{\sqrt{3}}{6}(x - 4)$ or $y = -\frac{\sqrt{3}}{6}x+\frac{5\sqrt{3}}{3}$.

45. **B** Recall the equation for solving for differentials: $f(x + \Delta x) = f(x) + f'(x)\Delta x$. In this case, $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2}x^{\frac{1}{2}}$, $x = 4$ and $\Delta x = 0.002$. If you plug all of those equations and values into the formula for differentials, the solution is 2.0005.

Section II

1. Water is dripping from a pipe into a container whose volume increases at a rate of The water takes the shape of a cone with both its radius and height changing with time.

(a) What is the rate of change of the radius of the container at the instant the height is 2 cm and the radius is 5 cm? At this instant the height is changing at a rate of 0.5 cm/min.

(a) The rate in the question stem refers to volume, so use equation for volume of a cone to relate radius and height. The volume of a cone is: $V = \frac{1}{3}\pi r^2h$. Differentiate this equation with respect to time to determine the rate of change of the radius: $\frac{dV}{dt}=\frac{1}{3}\pi\left(r^2\frac{dh}{dt}+2rh\frac{dr}{dt}\right)$. Now, plug in the given values, $r = 5$ cm, $h = 2$ cm, $\frac{dV}{dt} = 150$ cm^3/min, and $\frac{dh}{dt} = 0.5$ cm/min, so $150 = \frac{1}{3}\pi(5^2(0.5) + 2(5)(2)\frac{dr}{dt})$. Finally, solve for $\frac{dr}{dt}$: $\frac{dr}{dt} = 6.53697$ cm/min.

(b) The water begins to be extracted from the container at a rate of $E(t) = 75t^{0.25}$. Water continues to drip from the pipe at the same rate as before. When is the water at its maximum volume? Justify your reasoning.

(b) The rate of the volume of water now has to be adjusted because water is being extracted, so $\frac{dV}{dt} = 150 - E(t) = 150 - 75t^{0.25}$. To maximize the volume, set $\frac{dV}{dt}$ equal to 0 and solve for t: $150 - 75t^{0.25} = 0$, thus $t = 16$. To confirm this is a maximum, use the first derivative test. Since $\frac{dV}{dt} > 0$ when $0 < t < 16$ and $\frac{dV}{dt} < 0$ when $t > 16$, $t = 16$ is when the volume will be at a maximum. You can also go a step further and take the derivative of $\frac{dV}{dt}$ and use the second derivative test. $\frac{d^2V}{dt^2} = -\frac{75}{4}t^{-0.75}$ which is negative at $t = 16$, so $t = 16$ is a maximum.

(c) By the time water began to be extracted, 3000 cm^3 of water had already leaked from the pipe. Write, but do not evaluate, an expression with an integral that gives the volume of water in the container at the time in part (b).

(c) The volume of water in the container can be found by integrating the new expression for $\frac{dV}{dt}$ from part (b): $\frac{dV}{dt} = 150 - 75t^{0.25}$, over the interval $t = 0$ to $t = 16$, from part (b). In this part, we are given an initial volume that must be added to the volume found by the integral. Therefore the expression for the total volume is $V(t) = 3000 + \int_0^{16} 150 - 75t^{0.25}\, dt$.

2. The temperature in a room increases at a rate of $\frac{dT}{dt} = kT$, where k is a constant.

(a) Find an equation for T, the temperature (in °F), in terms of t, the number of hours passed, if the temperature is 65 °F initially and 70 °F after one hour.

(a) First, solve the differential equation for the rate of the temperature increase, $\frac{dT}{dt} = kT$, by separating the variables and integrating. When that is done, $T(t) = Ce^{kt}$. You are given that $T = 65$ at $t = 0$, so insert those values into the equation for T and solve for C: $65 = Ce^{k(0)}$, $C = 65$. Finally, you are given a second temperature and time point, (1,70); use those values in the new formula for T, $T(t) = 65e^{kt}$, and solve for k: $70 = 65e^{k(1)}$, $k = \ln\frac{14}{13} = 0.07411$. Therefore, the equation for T is $T(t) = 65e^{0.07411t}$.

(b) How many hours will it take for the temperature to reach 85 °F?

(b) Use the formula from part (a): $T(t) = 65e^{0.07411t}$. Insert 85 for $T(t)$ and solve for t: $85 = 65e^{0.07411t}$, thus $t = 3.6199$ hours.

(c) After the temperature reaches 85 °F, a fan is turned on and cools the room at a consistent rate of 7 °F/hour, how long will it take for the room to reach 0 °F?

(c) With the introduction of the fan cooling the room, the rate the temperature increases changes to $\frac{dT}{dt} = kT - 7$. Solve this differential equation by separating the variable and integrating: $T(t) = Ce^{kt} + \frac{7}{k}$. At $t = 0$, the temperature is 85 °F, so $C = 85 - \frac{7}{k}$. Use the value of k from part (a), $k = 0.07411$ and solve the equation for t when $T = 0$ °F: $0 = \left(85 - \frac{7}{0.07411}\right)e^{0.07411t} + \frac{7}{0.07411}$. Therefore, the time to get to 0 °F is 31.0548 hours.

3. Let R be the region enclosed by the graphs of $y = \frac{2}{x+1}$, $y = x^2$, and the lines $x = 0$ and $x = 1$.

(a) Find the area of R.

(a) First, determine which curve is more positive ($f(x)$), and set up your integral for area between curves: $A = \int_a^b (f(x) - g(x))dx$. For this problem, $f(x) = \frac{2}{x+1}$, so the integral is: $A = \int_0^1 \left(\frac{2}{x+1} - x^2\right)dx = 2\ln|2| - \frac{1}{3} \approx$ 1.05296.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(b) You can use the washer method to find the volume: $V = \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$. Thus, $V = \pi\int_0^1 \left[\left(\frac{2}{x+1}\right)^2 - (x^2)^2\right]dx = 1.8\pi$.

(c) Set up, but do not evaluate, the expression for the volume of the solid generated when R is revolved around the line $x = 2$.

(c) Here, use the cylindrical shells method: $V = 2\pi\int_a^b x(f(x) - g(x))dx$. Adjust the axis of rotation since we are revolving around the line $x = 2$. Because, $x = 2$ is more positive than the x-axis, we set up the integral with $(2 - x)$, not x. Thus, the integral is: $V = 2\pi\int_0^1 (2-x)\left(\frac{2}{x+1} - x^2\right)dx$.

4. Consider the equation $x^3 + 2x^2y + 4y^2 = 12$.

(a) Write an equation for the slope of the curve at any point (x,y).

(a) Use implicit differentiation to find the first derivative which is the slope of the curve at any point (x,y):

$x^3 + 2x^2y + 4y^2 = 12$

$3x^2 + 2(x^2\frac{dy}{dx} + 2xy) + 8y\frac{dy}{dx} = 0$

$3x^2 + 2x^2\frac{dy}{dx} + 4xy + 8y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x^2 + 8y) = -3x^2 - 4xy$

$\frac{dy}{dx} = \frac{-3x^2 - 4xy}{2x^2 + 8y}$

(b) Find the equation of the tangent line to the curve at $x = 0$.

(b) First, plug in $x = 0$ to the original equation to solve for y: $0^3 + 2(0)^2y + 4y^2 = 12$, so $y = \sqrt{3}$. Now, plug $x = 0$ and $y = \sqrt{3}$ into the equation for slope from part (a): $\frac{dy}{dx} = \frac{-3(0)^2 - 4(0)(\sqrt{3})}{2(0)^2 + 8(\sqrt{3})} = 0$. Use the point-slope form of a line to get your equation for the tangent line to point $(0, \sqrt{3})$: $y - \sqrt{3} = 0(x - 0)$, so $y = \sqrt{3}$.

(c) If the equation given for the curve is the path a car travels in feet over t seconds, find $\frac{d^2y}{dx^2}$ at $(0, \sqrt{3})$ and explain what it represents with proper units.

(c) Use $\frac{dy}{dx}$ from part (a) to find $\frac{d^2y}{dx^2}$ via implicit differentiation. Do not simplify; immediately plug in 0 for x, $\sqrt{3}$ for y, and 0 for $\frac{dy}{dx}$, from part (b): $\frac{d^2y}{dx^2}$ represents the car's acceleration. At the position $(0, \sqrt{3})$, the acceleration is $-\frac{1}{16}$ ft/sec^2.

5. Water is filling at a rate of 64π in^3 into a conical tank that has a diameter of 36 in at its base and whose height is 60 in.

(a) Find an expression for the volume of water (in in^3) in the tank in terms of its radius.

(a) The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. The height and radius of a cone are constantly proportionate at any point, so given the values for the height and diameter, we can write: $\frac{h}{r} = \frac{60}{18}$, so $h = \frac{10}{3}r$. Thus, in terms of r, the volume of the water in the tank will be found from evaluating: $V = \frac{10}{9}\pi r^3$.

(b) At what rate is the radius of the water expanding when the radius is 20 in.

(b) We can differentiate the formula for volume from part (a) with respect to time. Then we can plug in the rate the volume is changing, $\frac{dV}{dt} = 64\pi$, and the radius given, 20 in. $\frac{dV}{dt} = \frac{10}{3}\pi r^2 \frac{dr}{dt}$, so $64\pi = \frac{10}{3}\pi(20^2)\frac{dr}{dt}$. Then, $\frac{dr}{dt} = \frac{6}{125}\frac{\text{in}}{\text{sec}}$.

(c) How fast in (in/sec) is the height of the water increasing in the tank when the radius is 20 in?

(c) In order to find how fast the height is changing, we must go back to the relationship between height and radius in part (a) and rewrite the formula for volume with respect to height, not radius. Thus, $r = \frac{3}{10}h$ and $V = \frac{3}{100}\pi h^3$. If we differentiate this equation with respect to time, as in part (b), the rate will be found from the equation $\frac{dV}{dt} = \frac{9}{100}\pi h^2 \frac{dh}{dt}$. We can use the relationship between radius and height to solve for the height when the radius is 20 in, so the height is $\frac{200}{3}$in. Plugging in this value for h and the given value of $\frac{dV}{dt} = (64\pi)$. The equation to evaluate is $64\pi = \frac{9}{100}\pi\left(\frac{200}{3}\right)^2\frac{dh}{dt}$. From this, $\frac{dh}{dt} = \frac{4}{25}$ in/sec.

6. If a ball is accelerating at a rate given by $a(t) = -64\frac{\text{ft}}{\text{sec}^2}$, the velocity of the ball is $96\frac{\text{ft}}{\text{sec}}$ at time $t = 1$, and the height of the ball is 100 ft at $t = 0$, what is

(a) The equation of the ball's velocity at time t?

(a) The velocity of the ball can be found by integrating the acceleration function. $a(t) = \frac{dv}{dt} = -64$. So, $\int dv = \int -64\,dt$ or $v = -64t + C$. Plug in the condition that the velocity is 96 at $t = 1$ to solve for C: $C = 160$, so $v(t) = -64t + 160$.

(b) The time when the ball is changing direction?

(b) The ball changes direction when the velocity is zero, but the acceleration is not. Set $v(t)$ equal to zero and solve for such times, t. $v(t) = -64t + 160 = 0$ when $t = \frac{5}{2}$. Since $a(t)$ is a constant, (–64), the ball is changing direction at 2.5 sec.

(c) The equation of the ball's height?

(c) To determine the equation of the ball's height, repeat the procedure in part (a), but integrate the velocity function to get the position function: $v(t) = \frac{dh}{dt} = -64t + 160$. So, $\int dh = \int(-64t + 160)dt$ or $h(t) = -32t^2 + 160t + C$. To find C, plug in the ball's height at time $t = 0$, 100 ft, and $C = 100$. Thus $h(t) = -32t^2 + 160t + 100$.

(d) The ball's maximum height?

(d) The maximum height occurs when the velocity of the ball is zero, i.e. when it is changing direction from rising to falling. In part (b), we found that time to be $t = 2.5$ sec. Plug 2.5 into the position function to solve for the maximum height: $h(2.5) = -32(2.5)^2 + 160(2.5) + 100 = 300$ ft.