

Answers and Explanations

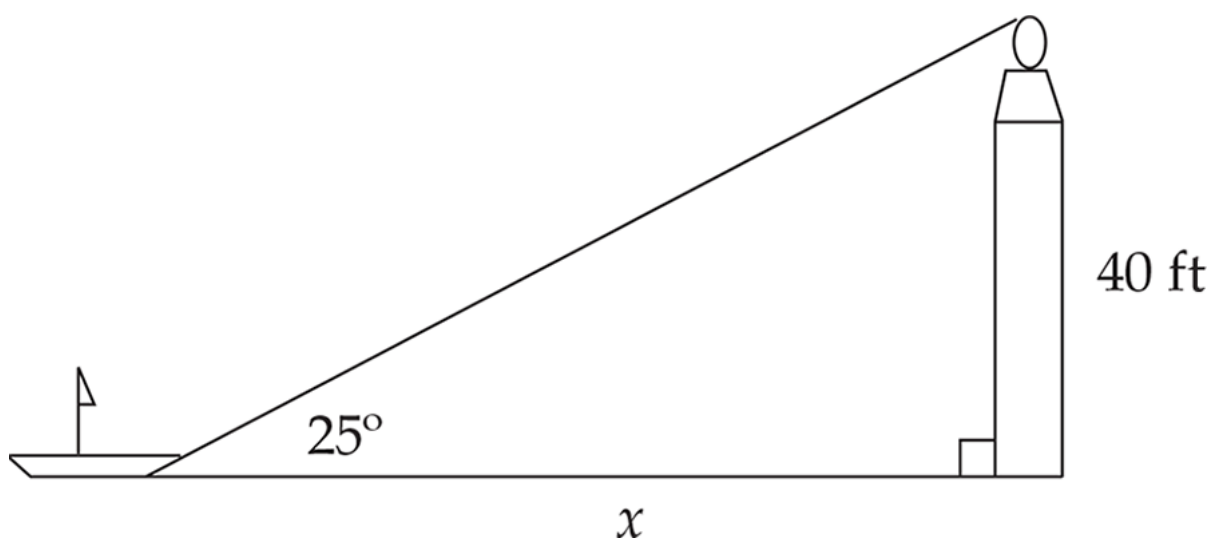
PRACTICE TEST 4

1. **A**

First, eliminate choice (E), since it is a linear equation, not a quadratic equation. Then you could attempt to factor each equation to determine the roots, then check the sum and the product of these roots. However, you may recall that in a quadratic equation in the form $ax^2 + bx + c$, the sum of the roots is $-\frac{b}{a}$, and the product of the roots is $\frac{c}{a}$. In choice (A), $-\frac{b}{a} = \frac{-2}{2} = -1$, and $\frac{c}{a} = \frac{-40}{2} = -20$.

2. **D**

Draw a picture of the problem situation:



As you can see from the drawing, this is a right triangle, where x represents the horizontal distance of the boat to the lighthouse. Use the tangent function to find the distance. The tangent of the angle is equal to the ratio of the lengths of $\frac{\text{opposite side}}{\text{adjacent side}}$. Set up the equation $\tan(25^\circ) = \frac{40}{x}$. Multiply both sides by x and then divide both sides by $\tan(25^\circ)$ to get $x = \frac{40}{\tan(25^\circ)} = 85.8$, to the nearest tenth. Use your calculator to determine this, making sure it's in degree mode.

3. **B**

The figure shows the geometric equivalent of a perfect square, broken up into a quadratic with pieces $x^2 + 7x + 7x + 49 = 121$, the given area. Factor the quadratic to get $(x + 7)(x + 7) = 121$, or $(x + 7)^2 = 121$. Take the square root of both sides of the equation to get $x + 7 = \pm 11$. Eliminate the root -11 , since the length of the side of a square cannot be negative. Subtract 7 from both sides of this equation to get $x = 4$.

4. **E**

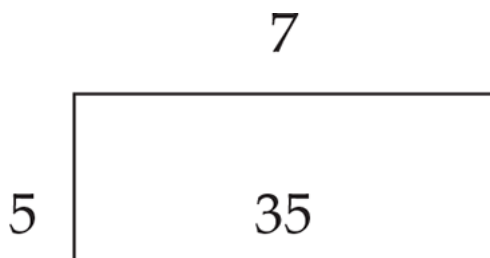
The company charges a flat fee of \$42.50, plus x dollars times the number of hours. The problem asks for the cost of a 90-minute call, which is 1.5 hours. The correct expression is therefore $42.50 + 1.5x$.

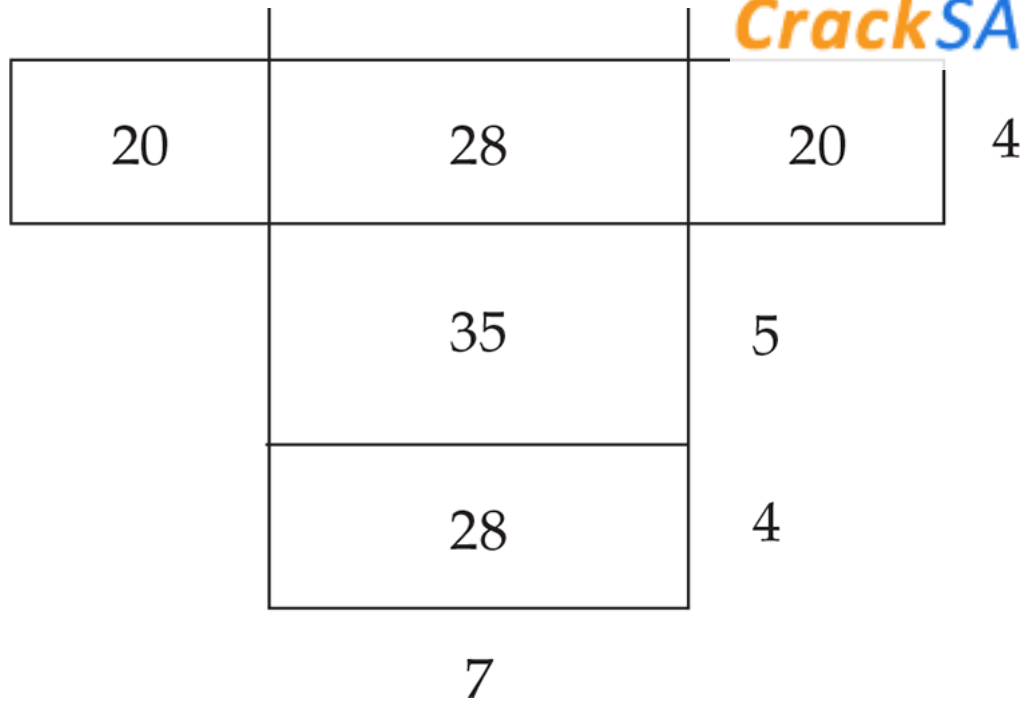
5. **A**

To solve this equation, express both sides as powers of the same base, in this case 4: $(4^3)^x = 4^{x^2-4}$. Use the power rule for exponents to get $4^{3x} = 4^{x^2-4}$. Since both sides of the equation are powers of 4, simplify the equation to be $3x = x^2 - 4$, or $x^2 - 3x - 4 = 0$. Factor the left-hand side to get $(x - 4)(x + 1) = 0$. Therefore, either $x = 4$ or $x = -1$. Alternatively, you could arrive at the correct answer by substituting in all the answer choices to find the correct solutions.

6. **B**

To solve this problem, sketch an "unfolded" drawing of this solid:





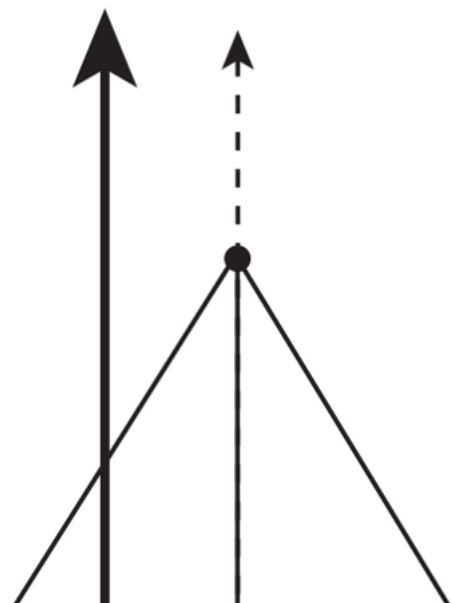
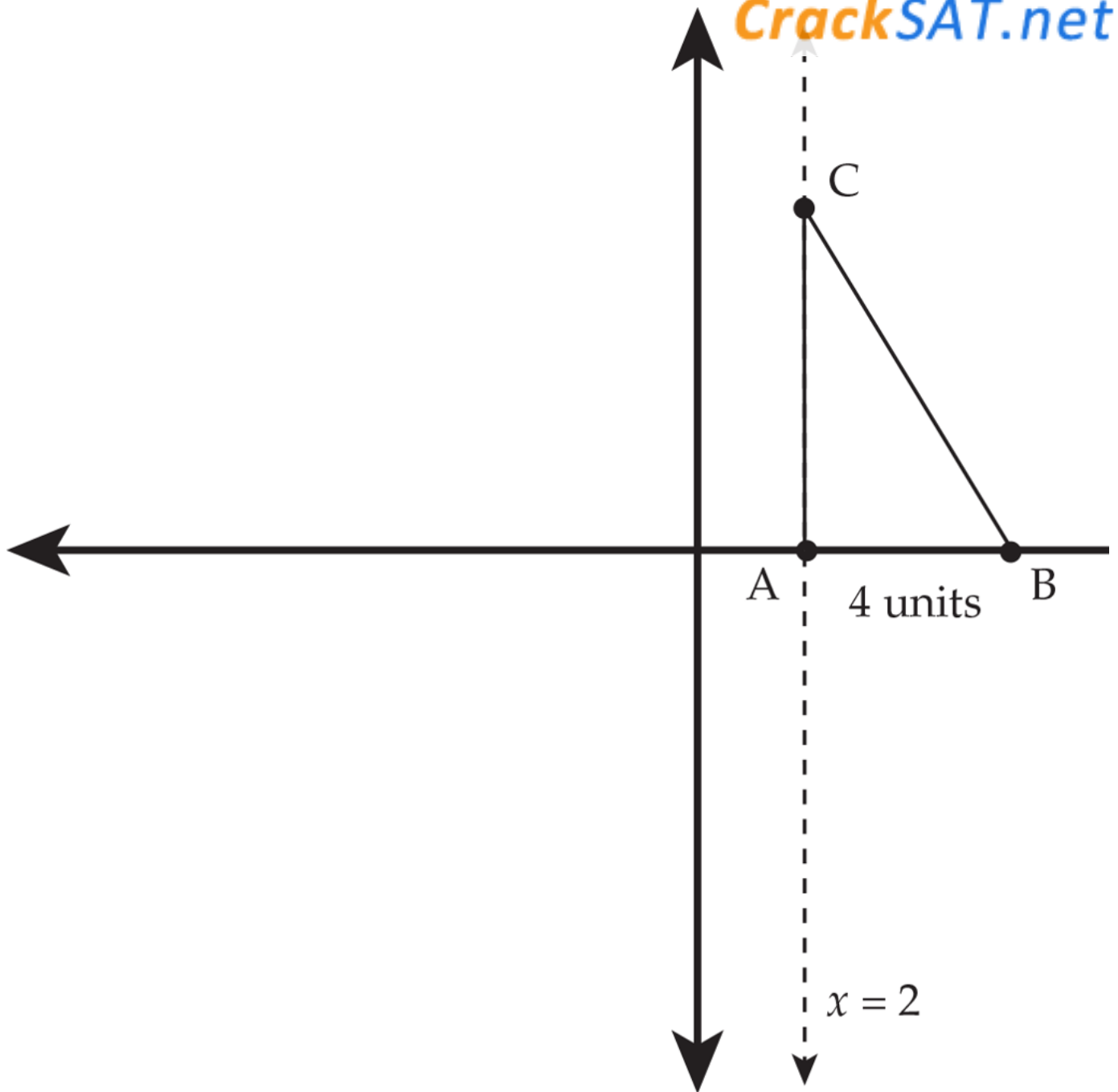
Opposite faces of a rectangular solid are congruent, so there are six faces, with the corresponding areas shown above. Look for factors that will produce the given areas. These factors are the dimensions of the solid: $7 \times 4 = 28$, $7 \times 5 = 35$, and $4 \times 5 = 20$. The dimensions are 7, 4, and 5. Use the volume formula to arrive at the correct answer: $V = lwh$, so $V = 7 \times 5 \times 4 = 140$ cubic centimeters.

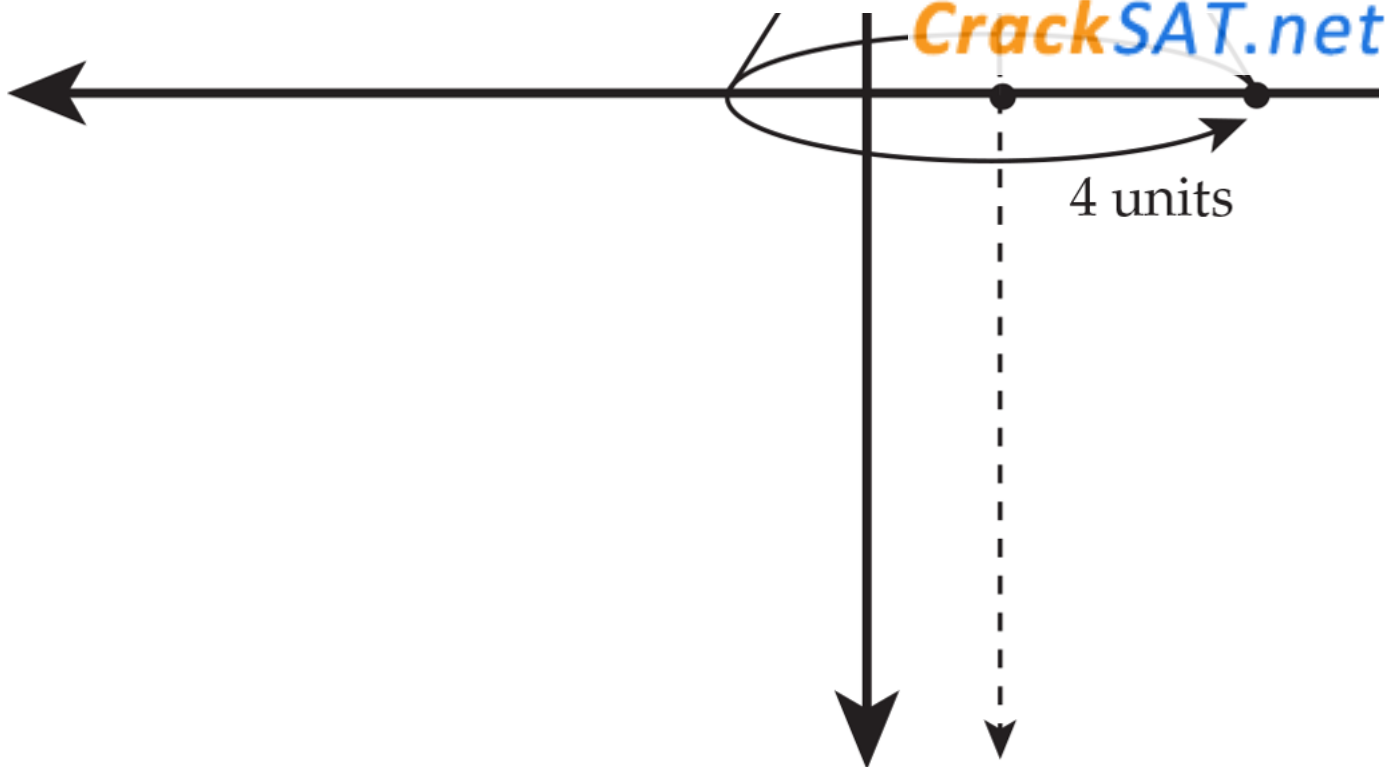
7. **B**

You can solve this equation by rewriting it in exponential form: $12^x = 640$. At this point, you could try each value of x to arrive at the correct answer, rounded. Alternatively, use logarithm rules to solve: $\log(12^x) = \log 640$, and $x \log(12) = \log 640$. Divide both sides by the $\log 12$ to get $x = \frac{\log 640}{\log 12}$, which is 2.6, to the nearest tenth.

8. **D**

Draw the triangle on the coordinate plane, as shown.





This is a right triangle, with one side on the line $x = 2$. As shown, when this triangle is rotated about $x = 2$, the result is a cone. The radius of the cone is the distance from point A to point B, or $6 - 2 = 4$ units. The height of the cone is the distance from point A to point C, or $6 - 0 = 6$. Use the formula for the volume of a right circular cone, which will be given on the formula sheet: $V = \frac{1}{3} \pi r^2 h$, or

$$V = \frac{\pi \times 4^2 \times 6}{3} = 32\pi.$$

9. **D**

First, determine how many numbers are available with just the restriction on the first digit. For the first digit, there are 8 choices, and then for the remaining 6 digits, there are 10 choices. So the pool of available numbers is $8 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$, or 8,000,000. If phone numbers cannot start with the digit pattern 911, then this eliminates $1 \times 1 \times 1 \times 10 \times 10 \times 10 \times 10$ numbers, or 10,000. There are another 10,000 numbers starting with 411. So the actual number of available phone numbers is $8,000,000 - 20,000 = 7,980,000$.

10. **C**

Subtract 30 from both sides of the inequality to get $x^2 + 7x - 30 \geq 0$. Factor the left-hand side: $(x + 10)(x - 3) \geq 0$. In order for the inequality to be true, either both factors must be positive, or both factors must be negative. This will be true when $x \geq 3$ or $x \leq -10$.

11. **A**

Determine the equation of the line through the points (1,5) and (3,9). The slope is $\frac{9-5}{3-1} = \frac{4}{2}$, which is 2. Find the y-intercept, b : $5 = 2(1) + b$, so $b = 3$. The equation of the line is $y = 2x + 3$. The line perpendicular to this line has a slope that is the negative reciprocal, namely $-\frac{1}{2}$, and the y-intercept is -2, because the given point is (0,-2). Now, find the intersection of these lines.

You could graph these lines on a graphing calculator and use the intersect function.

Alternatively, set one equation equal to the other and solve for x . $2x + 3 = -\frac{1}{2}x - 2$. Add $\frac{1}{2}x$ and subtract 3 from both sides to get $\frac{5}{2}x = -5$. Multiply both sides by $\frac{2}{5}$, and $x = -2$. Use this value for x in the first equation to find the corresponding y value. $y = 2(-2) + 3$; $y = -1$. The intersection point is (-2,-1).

12. **E**

For an absolute value equation, first isolate the absolute value term by adding 9 to both sides of the equation: $|4x - 3| < 9$. Now, solve two inequalities, $4x - 3 < 9$ and $-(4x - 3) < 9$. The first inequality simplifies to $4x < 12$, or $x < 3$. The second inequality simplifies to $-4x + 3 < 9$, or $-4x < 6$. When you divide by -4, you switch the inequality symbol, so $x > -\frac{3}{2}$, or $x > -1.5$. By combining the two inequalities, the solution can be written as $-1.5 < x < 3$.

13. **B**

The multiplicative inverse is the reciprocal, which is $\frac{1}{5-i}$. You must then rationalize the denominator by multiplying by $\frac{5+i}{5+i}$:

$$\frac{1}{5-i} \times \frac{5+i}{5+i} = \frac{5+i}{25-i^2}$$

Since $i = \sqrt{-1}$, the inverse becomes

$$\frac{5+i}{25-(-1)} = \frac{5+i}{26}$$

14. **A**

Rewrite the equation to be $x^3 = 125$. Take the cube root of both sides to get $x = 5$.

15. **E**

To find the inverse of a function, first write the original equation as $y = 7x + 5$. Now, exchange the x and y values and then solve for y : $x = 7y + 5$. Subtract 5 from both sides of the equation, then divide both sides by 7 to get $f^{-1}(x) = \frac{x-5}{7}$.

16. **E**

Translate the words into algebra. Six less than a number n is $n - 6$. The equation is thus $(n - 6)^5 = 32$. Take the fifth root of each side to get $n - 6 = 2$. Add 6 to both sides of the equation, and $n = 8$.

17. **D**

A fraction equivalent to $\frac{3}{5}$ has the form $\frac{3x}{5x}$.

Translate the words to get the equation $\frac{3x+2}{5x+2} = \frac{5}{8}$.

Solve for x by cross multiplying: $8(3x + 2) = 5(5x + 2)$. Use the distributive property to get $24x + 16 = 25x + 10$. Next, subtract $24x$ and 10 from both sides of the equation to get $x = 6$. Substitute this into the form of the original fraction:

$$\frac{3(6)}{5(6)} = \frac{18}{30}.$$

18. **A**

Express the first factor, with the common denominator of x , to get

$$\frac{x^2}{x} - \frac{3x}{x} - \frac{4}{x} = \frac{x^2 - 3x - 4}{x}.$$

Factor the numerator:

$$\frac{(x-4)(x+1)}{x}.$$

When this factor is multiplied by the second, the $(x+1)$ term will cancel to get

$$\frac{(x-4)}{x} \times \frac{3}{1} = \frac{3(x-4)}{x}.$$

19. **B**

Divide this polynomial using long division:

$$\begin{array}{r}
 3x^2 - 4x + 10 \\
 x + 2 \overline{) 3x^3 + 2x^2 + 2x + k} \\
 \underline{3x^3 + 6x^2} \\
 -4x^2 + 2x \\
 \underline{-4x^2 - 8x} \\
 10x + k \\
 \underline{10x + 20} \\
 k - 20
 \end{array}$$

The remainder is given as 4, so $k - 20 = 4$, and $k = 24$.

20. **B**

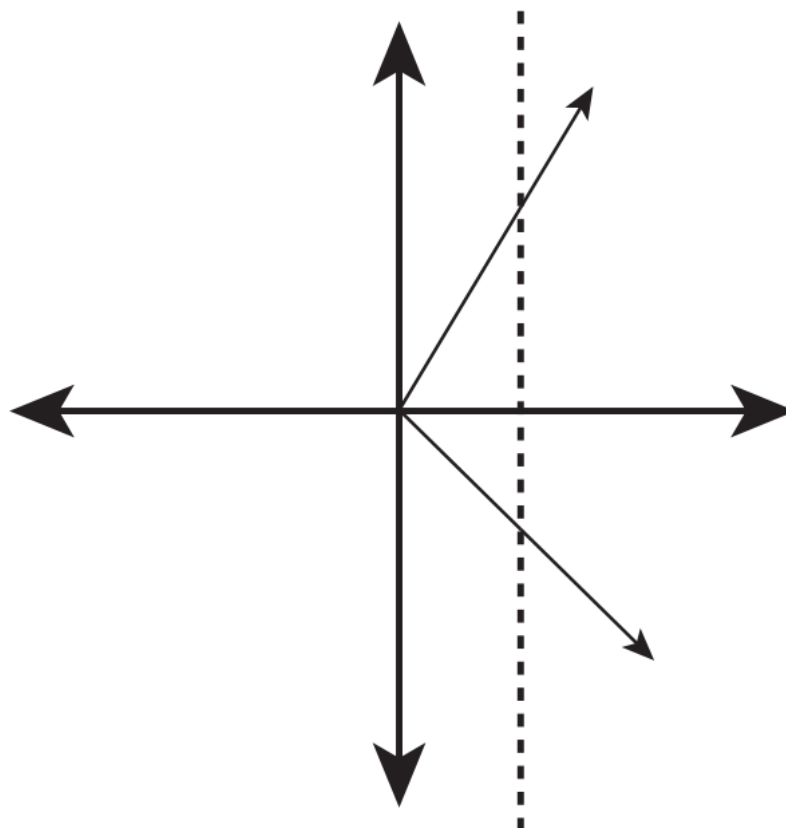
First, apply the power rule to the numerator: $(8x^2y^3)^3z^2 = 8^3x^{2 \times 3}y^{3 \times 3}z^2$. This simplifies to $512x^6y^9z^2$. Use the division rule of exponents on the numerator and the denominator to get $128x^{6-(-2)}y^{9-5}z^{2-4}$, which simplifies to $\frac{128x^8y^4}{z^2}$.

21. **C**

Solve the second equation for y by adding $3x$ to both sides to get $y = 3x + 1$. Now, set the two equations equal to each other: $3x + 1 = x^2 + 4x - 1$. Subtract $3x$ and 1 from both sides to get $x^2 + x - 2 = 0$. Factor the left-hand side: $(x + 2)(x - 1) = 0$. So $x = -2$, or $x = 1$. Replace x with the value -2 to find the corresponding y -coordinate: $-3(-2) + y = 1$, $6 + y = 1$, $y = -5$. So the first coordinate is $(-2, -5)$. To complete the calculation, replace x with the value 1: $-3(1) + y = 1$, $-3 + y = 1$, $y = 4$. So the other coordinate is $(1, 4)$. But note that, since the first coordinate appears in only one choice, you could have stopped after calculating it.

22. **D**

A function is a relation in which each x value has one unique corresponding y value. In graph D, the relation is not a function. Note the “vertical line test” in graph D below, which indicates that for a value of x , there are two possible y values.

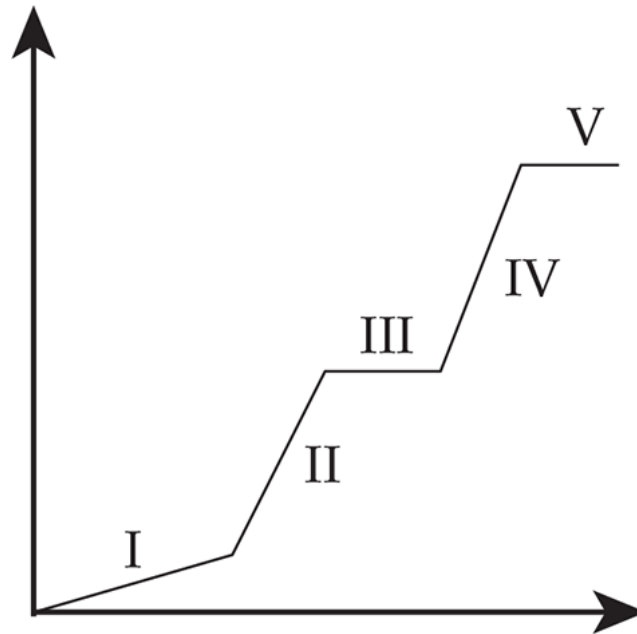


23. **E**

Solve the second equation for y by adding y and subtracting 5 from both sides to get $y = x - 5$. Now, substitute $x - 5$ for y in the first equation to get $x^2 + (x - 5)^2 = 25$. Multiply to get $x^2 + x^2 - 10x + 25 = 25$. Combine like terms and subtract 25 from both sides to get $2x^2 - 10x = 0$. Factor the left-hand side to get $2x(x - 5) = 0$. So $x = 0$ or $x = 5$. When $x = 0$, $y = 0 - 5 = -5$. When $x = 5$, $y = 5 - 5 = 0$. The points of intersection are $(0, -5)$ and $(5, 0)$.

24. **A**

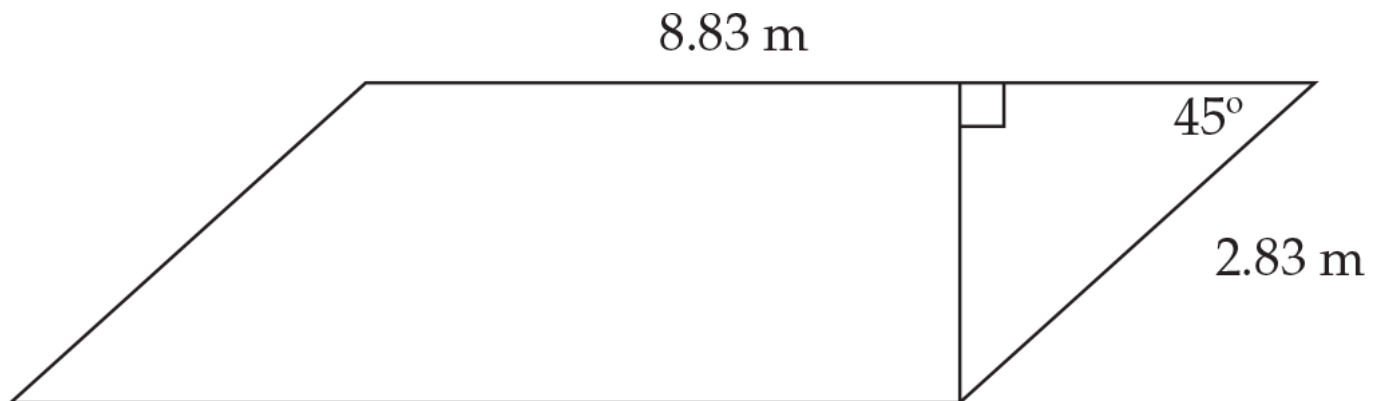
Graph A accurately describes the story.



The section marked I indicates the start of the trip. Section II represents the increase in speed; speed is the slope of the line. Section III is the standstill in traffic. Section IV denotes the continuation at the faster speed, and section V is the stop at point B.

25. **C**

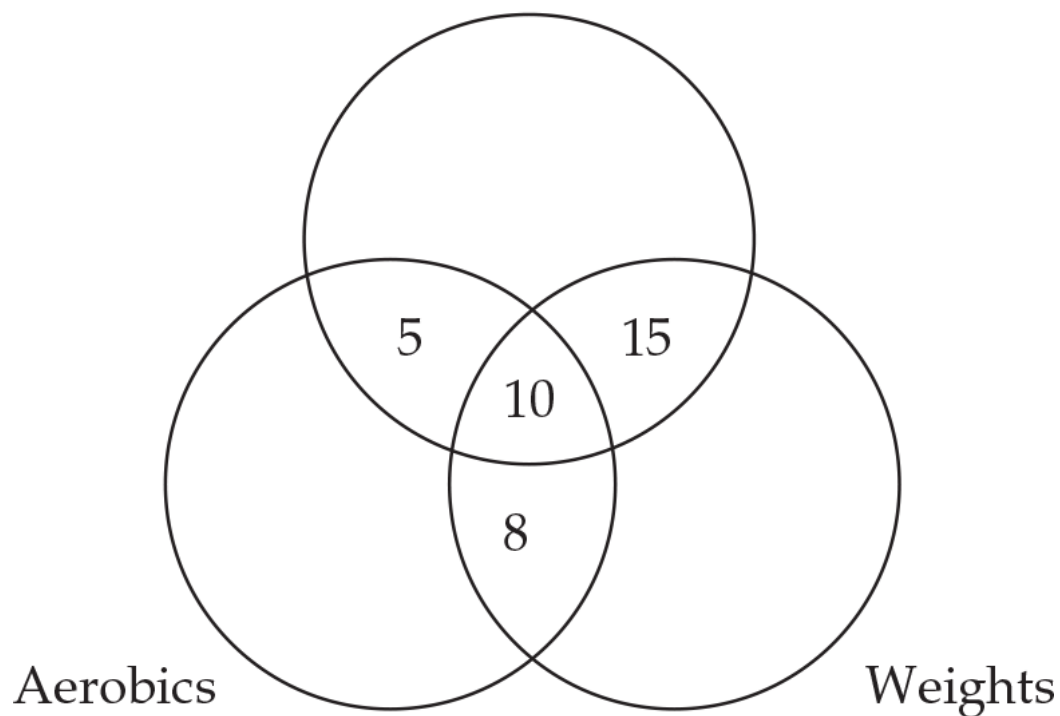
Draw the parallelogram with the height shown.



The triangle created inside the parallelogram is an isosceles triangle; two of the sides are congruent. In a 45–45–90 right triangle, if the congruent sides are x units long, then the hypotenuse is $x\sqrt{2}$. The hypotenuse is the shorter side of the parallelogram. It is given that the hypotenuse is 2.83; therefore, the height is $\frac{2.83}{\sqrt{2}} \approx 2.0$. The area of a parallelogram is base times the height, so the area $= 8.83 \times 2 \approx 17.67$, to the nearest hundredth.

26. **C**

It helps to draw a Venn diagram of the problem situation:



As shown in the diagram, add up the number of members who either do all three activities or any two of the activities: $8 + 10 + 5 + 15 = 38$. The remaining members use only one of the services, so $50 - 38 = 12$ members.

27. **C**

To solve the equation for x , first multiply both sides by $(x - a^2)$ to get $y(x - a^2) = x + 7a$. Use the distributive property on the left-hand side: $yx - ya^2 = x + 7a$. Add ya^2 and subtract x from both sides of the equation to get $yx - x = ya^2 + 7a$. Factor out the x from the left-hand side: $x(y - 1) = ya^2 + 7a$. Finally, divide both sides by $(y - 1)$ to solve for x in terms of y and a : $x = \frac{ya^2 + 7a}{y - 1}$.

28. **A**

First, raise both sides of the equation to the third power to eliminate the cube root on the left-hand side. This results in $\frac{2x+3}{5} = \frac{2^3}{3^3}$, or $\frac{2x+3}{5} = \frac{8}{27}$. Cross multiply to get $27(2x + 3) = 40$, or $54x + 81 = 40$. Subtract 81 from both sides and divide both sides by 54 to get $x = -\frac{41}{54} = -0.76$.

29. **A**

It is given that $k = 21$, so substitute this value in to find the value of y :

$$y = \frac{7}{k} = \frac{7}{21} = \frac{1}{3}.$$

Use this value of y in the first equation: $x = y^3 + 4y$, or

$$x = \left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right) = \left(\frac{1}{27}\right) + \left(\frac{4}{3}\right) =$$

30. **E**

Combine the left-hand terms by using a common denominator of $5x$:

$$\frac{5+4}{5x} = \frac{2}{x+5}, \text{ or } \frac{9}{5x} = \frac{2}{x+5}.$$

Cross multiply to get $9x + 45 = 10x$. Subtract $9x$ from both sides, and $x = 45$.

31. **D**

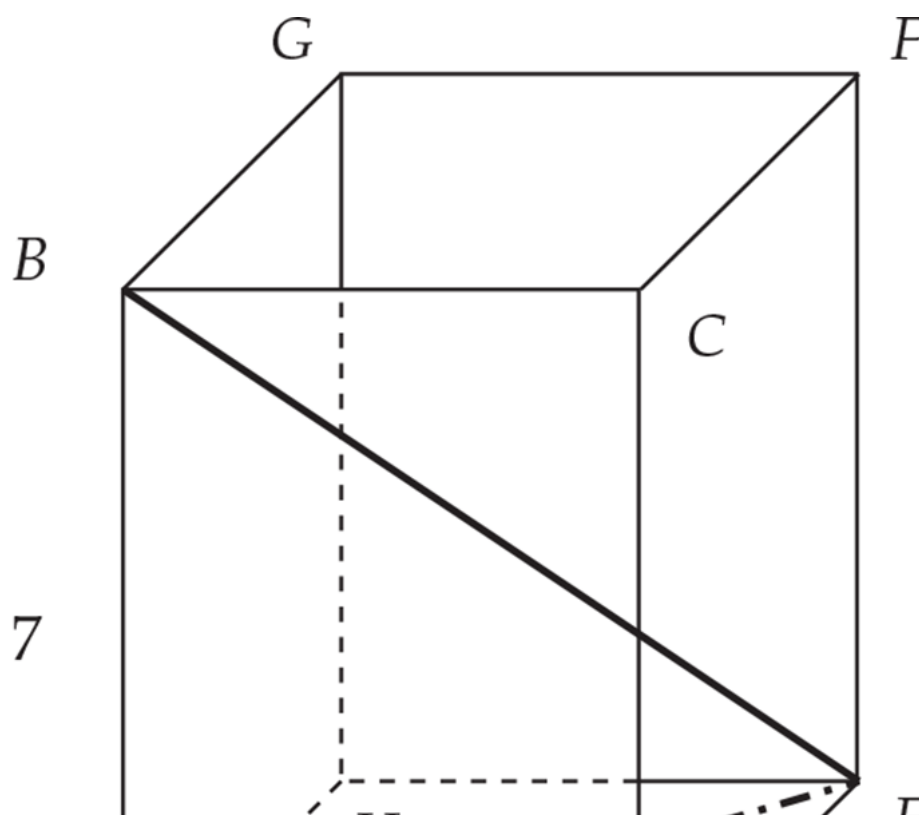
Let h represent the number of hours to drive 540 miles. Set up the proportion of miles over hours: $\frac{x}{1} = \frac{540}{h}$. Cross multiply to get $xh = 540$. Divide both sides by x to solve for h , the number of hours, to get $h = \frac{540}{x}$.

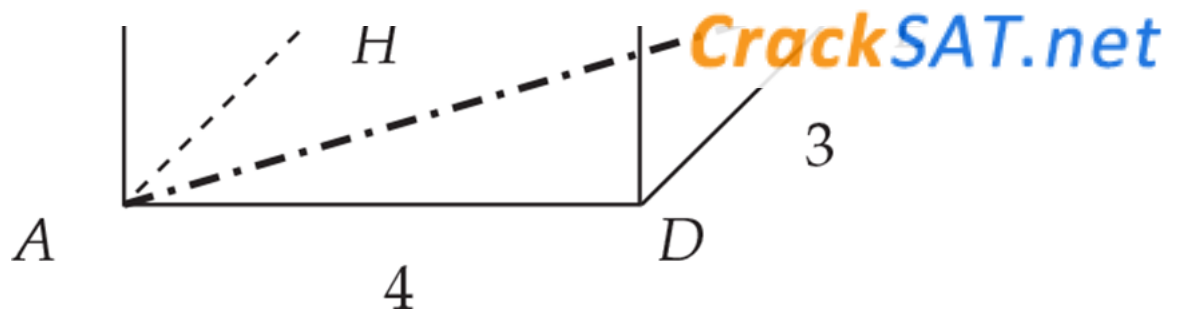
32. **A**

Let x represent the length of AB . Then translate the words to represent $BC = 2x + 3$. These segments added together equal AC , so $x + 2x + 3 = 27$. Combine

like terms and subtract 3 from both sides to get $3x = 24$; divide both sides by 3 to get $x = 8$.

33. **E**
In a composition of functions, first evaluate the innermost function, in this case $g(2)$: $4 \times 2 + 3 = 8 + 3 = 11$. Now use this result in the outermost function f : $f(11) = 11^2 - 10 = 121 - 10 = 111$.
34. **C**
(A) and (D) can be immediately eliminated because the boundary lines are solid. The inequalities stated in the problem are "less than" and "greater than," which denote dotted boundary lines. (B) can be eliminated because one of the boundary lines in this graph is a horizontal line. This would be $y < 2$, not $y < x + 2$. (E) is incorrect because the shading would indicate two "less than" inequalities; the shading is below both of the boundary lines.
- (C) is the only graph that could represent the system. The first inequality is represented by a dotted line through the y -intercept of 2 with a slope of 1. It is then shaded below this line since the inequality is "less than." The second inequality is represented by a dotted line through the y -intercept of 3 with a slope of -3 . It is then shaded above this line since the inequality is "greater than." Where the shadings meet is the solution to the system of inequalities, or graph (C).
35. **B**
The problem is asking for the length of the diagonal through the rectangular solid. First, determine the length of the diagonal through the bottom base of the solid. This is shown as the bold dotted line (segment AE).





This is the hypotenuse of a right triangle with legs of length 3 and 4. Use the Pythagorean theorem: $a^2 + b^2 = c^2$, or $3^2 + 4^2 = c^2$. This diagonal is therefore

$$\sqrt{(9 + 16)} = \sqrt{25} = 5.$$

(It's also a special 3-4-5 right triangle.) Now, the diagonal of the solid is the hypotenuse of the right triangle with legs of length 5 and 7. In the figure, segment AE is now the leg of a second right triangle, whose hypotenuse is the length to be solved for. Use the Pythagorean theorem: $5^2 + 7^2 = c^2$;

$$c = \sqrt{(25 + 49)} = \sqrt{74} \approx 8.6.$$

36. **B**

Because you are given the ratio of AB to AC , first set up a proportion to find the length of AC in terms of x : $\frac{5}{8} = \frac{x}{AC}$. Cross multiply to get $5(AC) = 8x$, or $AC = \frac{8x}{5}$. Divide this result by 2 since the problem asks for the midpoint of $\overline{AC}$, which is therefore $\frac{8x}{10} = \frac{4x}{5}$.

37. **E**

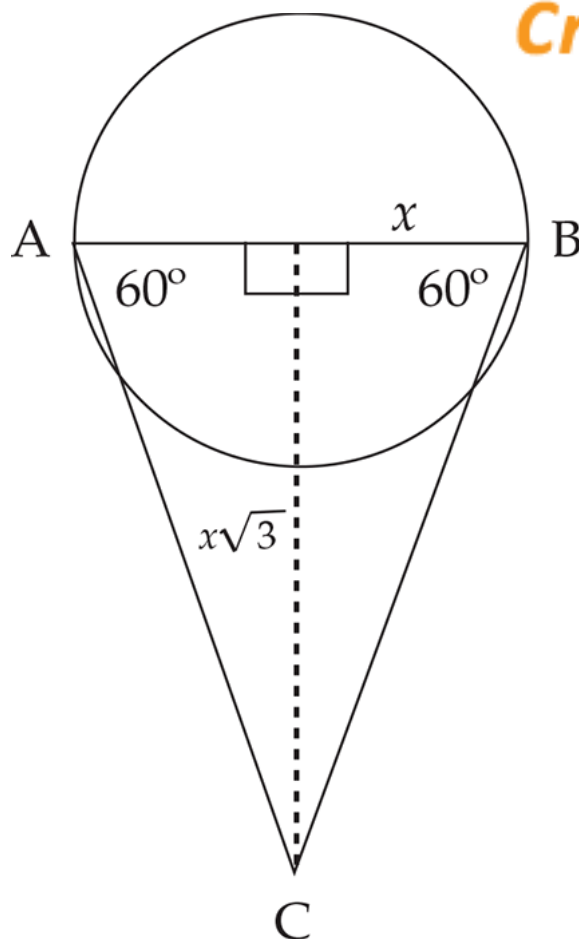
Substitute the value of 8 for a and 3 for b to get $8^2 + 8 \times 3 - 16 \times 3 \div 8$. Now, use the order of operations to simplify. First, take 8 squared (64), and then perform multiplication and division from left to right. Multiply to get $64 + 24 - 48 \div 8$. Next perform division: $48 \div 8 = 6$. Then, first add and finally subtract: $64 + 24 - 6 = 82$.

38. **D**

To find the area of the shaded region, find the area of the square, then subtract out the area of the circle. The square's diagonal length is given as 19.8 m. This is also the hypotenuse of an isosceles right triangle, and the legs of the triangle are the congruent sides of the square. In a 45-45-90 triangle, the hypotenuse is $\sqrt{2}$ times as long as the legs, so the leg length is $\frac{19.8}{\sqrt{2}} \approx 14$ m. Since the circle is inscribed in the square, the radius is one-half the length of a side, or 7 m. Find the area by calculating $A_{\text{square}} - A_{\text{circle}}$, or $s^2 - \pi r^2$; $14^2 - \pi(7^2)$; $196 - 153.94 = 42.06$.

39. **B**

Use the given area of the circle, 64π , to find the radius, r : $r^2 = 64$, so $r = 8$. This radius is one-half the base of the equilateral triangle shown, so the base is 16. As shown in the figure below, there are two 30-60-90 right triangles.



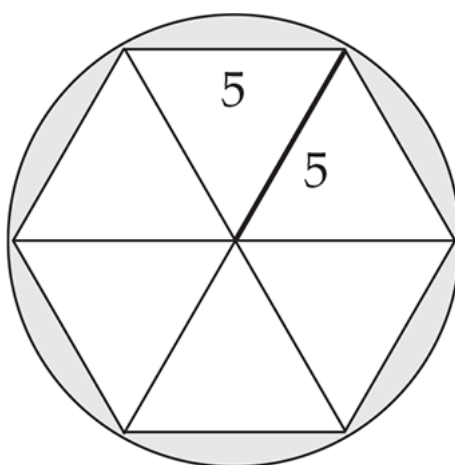
In this type of triangle, the legs are x and $x\sqrt{3}$. Therefore, the height of the triangle is $8\sqrt{3}$ units long. Now the area of the 30–60–90 triangle can be calculated:

$$A = \frac{1}{2} \times 8 \times 8\sqrt{3} \approx 55.43.$$

The equilateral triangle is two of these; the area is $2 \times 55.425 \approx 110.85$.

40. **B**

A regular hexagon is made up of six equilateral triangles, in this case with length of 5 units:



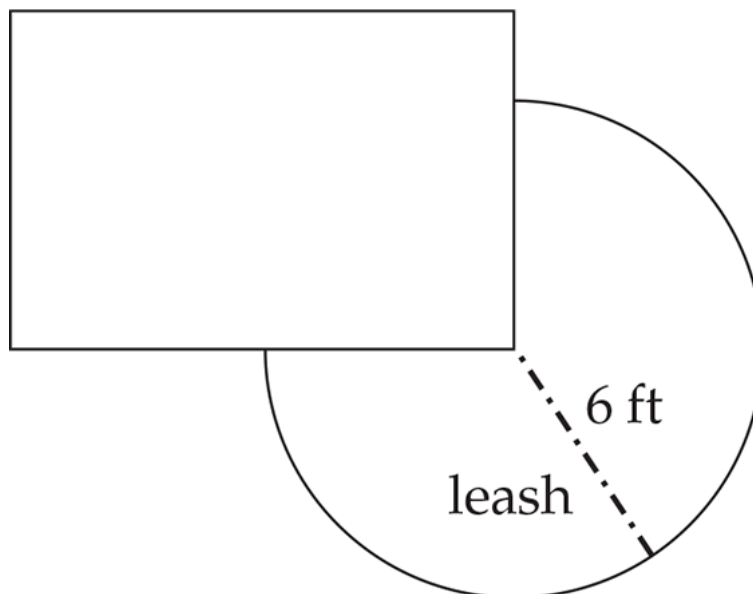
As shown above, the radius is therefore 5 units. The area of the circle is $A = \pi r^2 = 25\pi$. The area of a hexagon with side length x is given as $3 \times x \times \frac{x}{2} \sqrt{3}$. The shaded portion is $\text{Area}_{\text{circle}} - \text{Area}_{\text{hexagon}}$, or

$$25\pi - 3(5)(2.5)(\sqrt{3}) \approx 13.588.$$

The percentage of the shaded portion can be found by dividing this area by the area of the circle: $13.588 \div 25\pi \approx 0.173$, or 17.3%.

41. **D**

Draw a picture of the situation to solve the problem:



As shown in the diagram above, the dog has $\frac{3}{4}$, or 0.75, of a circle in which to move. The circle has a radius equal to the length of the chain, 6 ft. The area the dog has to move in is therefore $0.75(\pi)(6^2) \approx 84.82 \text{ ft}^2$.

42. **B**

The domain of this function would be the real numbers, with the exception of any numbers that would produce a zero in the denominator or a negative value within the radicand. The value of 4 will produce a zero in the denominator; any number greater than 4 will be part of the domain. In the same way, because $(-4)^2 = 16$, any number less than -4 will be part of the domain.

43. **B**

Using the stem-and-leaf plot, calculate the mean, median, and mode to determine the truth of the three statements. The mode is 22, the value that occurs most often. The median is the middle value. In a stem-and-leaf plot, the values are ordered. There are twenty values in the plot, so the median is halfway between the tenth and eleventh value, or halfway between 33 and 35. The median is 34. The mean is the sum of the values divided by 20, or $(22 + 22 + 22 + 22 + 24 + 27 + 31 + 32 + 32 + 33 + 35 + 36 + 37 + 45 + 47 + 47 + 50 + 51 + 51 + 51) \div 20$, or $717 \div 20 = 35.85$. The only true statement is II, that the median (34) is less than the mean (35.85).

44. **D**

Out of the 11 possible cards, 2 of them are M's. The probability of choosing an M is therefore $\frac{2}{11}$. One card is now gone; there are 10 possible cards and 2 of them are A's. This probability of picking an A is $\frac{2}{10} = \frac{1}{5}$. The probability of this compound event is $\frac{2}{11} \times \frac{1}{5} = \frac{2}{55}$.

45. **A**

Out of the 11 possible cards, 1 of them is an S. The probability of choosing an S is therefore $\frac{1}{11}$. One card is now gone; there are 10 possible cards and 2 of them are T's. So 8 of them are *not* T's. The probability of not drawing a T is $\frac{8}{10} = \frac{4}{5}$. The probability of this compound event is $\frac{1}{11} \times \frac{4}{5} = \frac{4}{55}$.

46. **C**

This is a calculator problem. The problem asks for the measure of an acute angle of a right triangle. Use trigonometry to solve. Side $\overline{DF}$ is adjacent to $\angle EFD$, and side $\overline{EF}$ is the hypotenuse. Use the cosine function: $\cos(\angle EFD) = \frac{2}{7}$, so the measure of

$$\angle EFD = \arccos\left(\frac{2}{7}\right),$$

which is 73.40° , to the nearest hundredth of a degree.

47. **D**

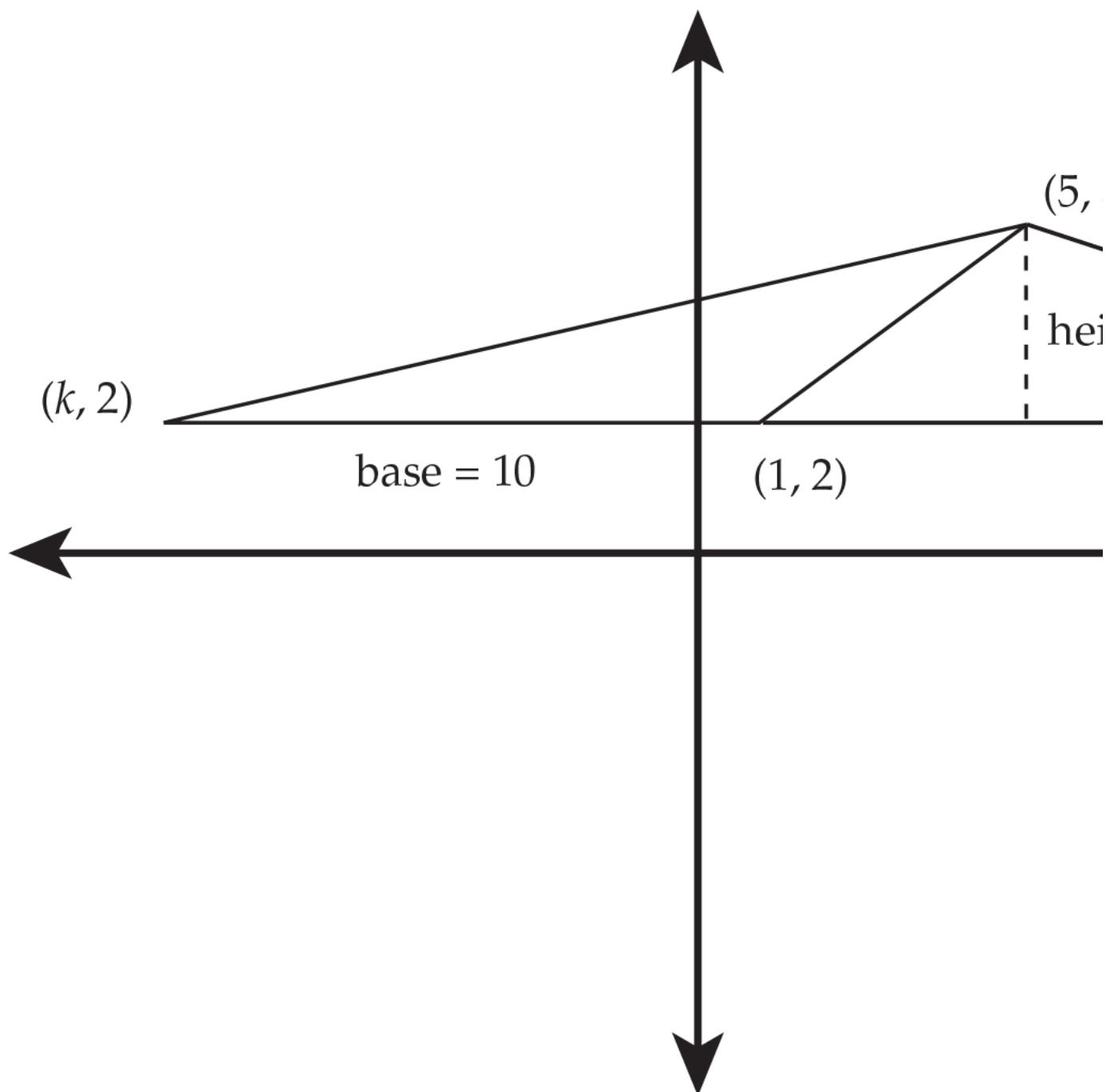
Substitute the value of $(a - 3)$ for x in the function: $f(a - 3) = (a - 3)^2 - 7$. Multiply to get $(a - 3)(a - 3) - 7 = a^2 - 6a + 9 - 7$. Combine like terms: $a^2 - 6a + 2$.

48. **A**

The coordinates of the midpoint of two points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Setting $x_1 = -3$ and $y_1 = -8$, solve for x_2 and y_2 . First, $1 = \frac{-3 + x_2}{2}$, or $2 = -3 + x_2$. Add 3 to both sides, and $x_2 = 5$. Second, $-5 = \frac{-8 + y_2}{2}$, or $-10 = -8 + y_2$. Add 8 to both sides to get $y_2 = -2$.

49. **B**

Draw the triangle on a coordinate plane, as shown below:



The formula for the area of a triangle is $A = \frac{1}{2}bh$. Because the area is given as 15 square units, and the height is $5 - 2 = 3$ units, the base must be 10 units long. Refer to the drawing above and see that the length for the base is the difference between the x-coordinate of (1,2) and the x-coordinate of (k,2). The base can therefore be represented as $k - 1$, or $1 - k$. Therefore, $k = 11$, or $k = -9$.

50. **A**

The problem shows a right triangle in which an angle measure and the length of the hypotenuse are known. Use trigonometry to find the length of $\overline{ST}$, the opposite side of the 42° angle. Using x to represent the length of $\overline{ST}$,

$$\sin(42^\circ) = \frac{x}{18}.$$

Multiply both sides by 18, and $x = 18 \times \sin(42^\circ)$. $x = 12.04$ to the nearest hundredth.

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