The **ISEE** Course Book

Focusing on the Individual Student

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Answer Key

Mathematics Achievement

- The Mathematics Achievement section measures your knowledge of math skills, concepts, procedures, and terminology.

- Mathematics Achievement questions conform to national mathematics standards, so they are similar to the typical questions seen in school math courses. The questions require one or more steps in calculating the answer. In order to solve these problems, you are expected to use standard math rules and processes. Incorrect answers are usually the result of student errors, such as miscalculations or using the wrong formulas.

- Since each level of the ISEE is given to students in more than one grade, some of the questions may seem especially difficult because you have not yet learned some of the concepts. However, your score on the ISEE is only compared to students in the same grade as you, so these other students have probably learned no more than you have.

- Directions are as follows:

> Each question is followed by four suggested answers. Read each question and then decide which one of the four suggested answers is best.
> Find the row of spaces on your answer document that has the same number as the question. In this row, mark the space having the same letter as the answer you have chosen. You may write in your test booklet.
>
> Which expression is equivalent to $3 \times (4 + 5)$?
>
> (A) $3 + 9$
> (B) 3×9
> (C) $12 + 5$
> (D) 7×5
>
> Sample Answer
>
> 
>
>
> The correct answer is 3×9, so circle B is darkened.

- Depending on your test level, the Mathematics Achievement section will have different numbers of certain question types.

Lower Level Mathematics Achievement

Whole Numbers	4-7 questions
Decimals / Percents / Fractions	4-7 questions
Algebraic Concepts	4-7 questions
Geometry	2-5 questions
Measurement	2-5 questions
Data / Analysis / Probability	4-7 questions

Middle Level Mathematics Achievement

Whole Numbers	7-10 questions
Decimals / Percents / Fractions	7-10 questions
Algebraic Concepts	9-13 questions
Geometry	4-6 questions
Measurement	4-6 questions
Data / Analysis / Probability	5-9 questions

Upper Level Mathematics Achievement

Number Sense	5-11 questions
Algebraic Concepts	13-17 questions
Geometry	5-8 questions
Measurement	5-8 questions
Data / Analysis / Probability	8-13 questions

Solving Backwards

- Many word problems can be solved by using the answer choices to work backward. This may be easier than trying to set up an algebraic equation.

Out of 73 black and white socks in a drawer, there are 23 more white socks than there are black socks. How many white socks are there?

(A) 25
(B) 45
(C) 48
(D) 50

Now go to the answer choices.

Let's try (B).

If there are 45 white socks, there are 22 black socks (45 – 23 = 22). This gives a total of 67 socks. Since 67 is less than 73, we know there must be more than 45 white socks. So your choices are (C) or (D).

Let's try (D).

If there are 50 white socks, there are 27 black socks (50 – 23 = 27). This gives 77 total socks. Incorrect.

The answer must be (C), but let's check it.

48 white socks means there are 25 black socks (48 – 23 = 25).
48 + 25 = 73. (C) is correct.

- Use the answer choices as a guide for how to solve questions.

- Some questions are so complicated that it is hard to put all of the information together. It often helps to look at what the question is asking you for and then to consider what you need to do to find that information.

TRY IT OUT

Use the answer choices to solve the following word problems:

1. There are 5 times as many pairs of white socks as there are pairs of black socks in Jason's drawer. If the total number of pairs of socks equals 36, then how many pairs of white socks are in the drawer?

   (A) 6
   (B) 15
   (C) 18
   (D) 30

---

2. George and Bessie both collect stamps. If George has 22 stamps and Bessie 38 stamps, how many stamps must George purchase from Bessie if they are to have the same number of stamps in their collections?

   (A) 4
   (B) 8
   (C) 16
   (D) 18

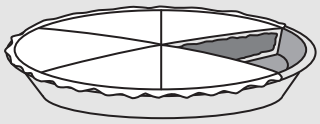
---

3. 24 feet of fence enclose Dan's rectangular yard. What's the yard's width, if the width is $\frac{1}{3}$ of the length?

   (A) 3 feet
   (B) 6 feet
   (C) 9 feet
   (D) 18 feet

Fractions

- Fractions are used to represent portions of a whole. Fractions are written as $\frac{\text{PART}}{\text{WHOLE}}$. Fractions also represent divisions, with the part divided by the whole.



The pie above is divided into 6 equal parts.

One part, or $\frac{1}{6}$ of the pie, is missing. Five parts, or $\frac{5}{6}$ of the pie, are left.

- The **numerator** is the top part of a fraction. The **denominator** is the bottom part.

- An **improper fraction** has a numerator that is greater than or equal to the denominator.

$\frac{3}{2}$ $\frac{11}{8}$ $\frac{9}{4}$ $\frac{7}{7}$

- A **mixed number** is made up of a whole number and a fraction. It is equal to the sum of the whole number and the fraction.

$1\frac{1}{2}=1+\frac{1}{2}$ $1\frac{3}{8}=1+\frac{3}{8}$ $2\frac{1}{4}=2+\frac{1}{4}$

There are 7 slices of pie and each slice is $\frac{1}{6}$ of a whole pie.

This can be expressed as $\frac{7}{6}$ or as $1\frac{1}{6}$ pies.

- To convert a whole number to a fraction, first determine the value of the denominator. The numerator will be the product of the whole number and the denominator.

Convert 3 to sixths: $3 \times 6 = 18 \Rightarrow 3 = \frac{18}{6}$

Convert 7 to fourths: $7 \times 4 = 28 \Rightarrow 7 = \frac{28}{4}$

$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4}$ $\quad 2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4}$ $\quad 3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4}$

$10 = \frac{10}{1} = \frac{20}{2} = \frac{30}{3} = \frac{40}{4}$ $\quad 100 = \frac{100}{1} = \frac{200}{2} = \frac{300}{3} = \frac{400}{4} = \frac{500}{5}$

- To convert a mixed number to an improper fraction, rewrite the whole number as a fraction with the same denominator. Then, add the two fractions.

Convert $2\frac{3}{5}$ to an improper fraction:

$2\frac{3}{5} \Rightarrow 2 + \frac{3}{5} \Rightarrow \frac{10}{5} + \frac{3}{5} \Rightarrow \frac{13}{5}$

- To convert an improper fraction to a mixed number, divide the numerator by the denominator. The whole number of the dividend will be the whole number and the remainder will be the numerator of the fraction.

$\frac{17}{5} \Rightarrow 5\overline{)17} \Rightarrow 3 \text{ R2} \Rightarrow 3\frac{2}{5}$

TRY IT OUT

Convert mixed numbers to improper fractions:

1. $1\frac{1}{4}$

2. $6\frac{2}{3}$

3. $5\frac{6}{7}$

Convert improper fractions to mixed numbers:

4. $\frac{3}{2}$

5. $\frac{8}{3}$

6. $\frac{19}{4}$

Solve:

7. If half a number is 5, what is the whole number?

8. If one third of a number is 4, what is the whole number?

9. If $\frac{4}{9}$ of a number is 7, what is $\frac{8}{9}$ of the number?

10. If $\frac{3}{4}$ of a number is 15, what is $\frac{1}{4}$ of the number?

11. If $\frac{4}{7}$ of a number is 40, what is the whole number?

12. If there are 300 calories in a $\frac{2}{3}$-cup serving of granola, how many calories are there in a 1-cup serving?

PUT IT TOGETHER QUESTIONS 1-2: LOWER LEVEL QUESTIONS 3-5: MIDDLE LEVEL

1. What is the value of the expression $\left(4\times\frac{1}{4}\right)+\left(3\times\frac{1}{3}\right)$?

   (A) $\frac{4}{7}$
   (B) 1
   (C) $\frac{7}{4}$
   (D) 2

---

2. When 30 gallons of water were poured into an empty tank, the water filled $\frac{2}{3}$ of the tank's total capacity. What is the tank's total capacity in gallons?

   (A) 20
   (B) 45
   (C) 60
   (D) 75

---

3. If n is greater than 1, then $\frac{1}{n}$ must be

   (A) less than 1
   (B) greater than 1
   (C) greater than n
   (D) between 1 and n

---

4. If $\frac{2}{3}$ of a number is 18, then $\frac{4}{3}$ of the number is equal to which of the following?

   (A) 9
   (B) 27
   (C) 36
   (D) 72

---

5. A jug that is $\frac{3}{5}$ full contains 15 ounces of liquid. How many ounces of liquid does the jug hold when it's full?

   (A) 9
   (B) 10
   (C) 25
   (D) 45

Data Plots (Middle & Upper Level)

- "Stem and Leaf" plots are a way to visualize data by grouping numbers.

  A stem and leaf plot has a vertical line that typically separates the ones digits on the right (the "leaf") from the tens digits on the left (the "stem").

{2, 43, 17, 4, 21, 26, 15, 20, 17, 48, 27}

0	2, 4
1	5, 7, 7
2	0, 1, 6, 7
3	
4	3, 8

- "Box and Whisker" plots are a way to visualize data by organizing into quartiles.

  Begin by putting your values into numerical order, and then find the median. Next, find the median of the values smaller than the median. Also find the median of the values greater than the median. These three different medians will separate the data set into four groups, called "quartiles."

  The "Box and Whisker" plot shows the range of the data set, with the "box" around the middle two quartiles and the "whiskers" extending through the highest and lowest quartiles.

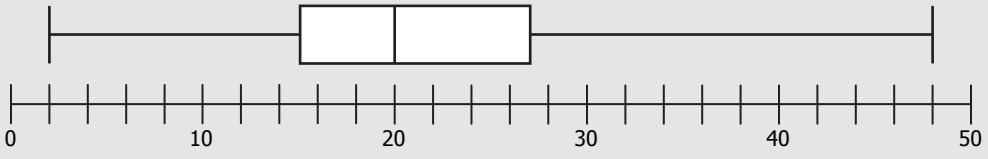
{2, 43, 17, 4, 21, 26, 15, 20, 17, 48, 27}

Put in order: {2, 4, 15, 17, 17, 20, 21, 26, 27, 43, 48}

median

Find upper & lower medians: {2, 4, 15, 17, 17}, 20, {21, 26, 27, 43, 48}

lower median upper median



TRY IT OUT

1. Find the median, range, and mode for the data represented by the "Stem and Leaf" plot:

2	1, 8
3	6, 7
4	5
5	3
6	0, 9, 9

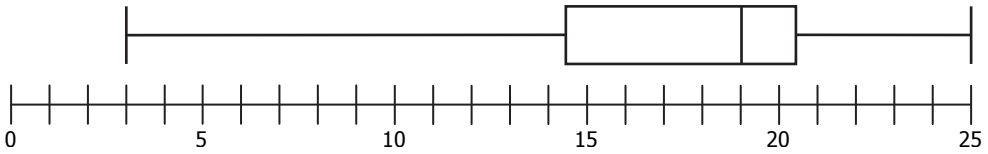
Median:

Range:

Mode:

---

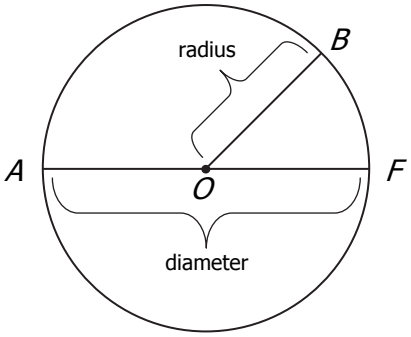
2. Find the median and range for the data represented by the "Box and Whisker" plot:



Median:

Range:

Circles



The **center** of the circle is located at *O*.

The **radius** of the circle ($\overline{OB}$) starts at the center and ends on the edge of the circle. All radii of a given circle have the same length.

The **diameter** of the circle ($\overline{AF}$) cuts directly through the center of the circle and is twice the radius. All diameters of a given circle have the same length.

- **Pi** (π), is the ratio of a circle's diameter to its circumference.

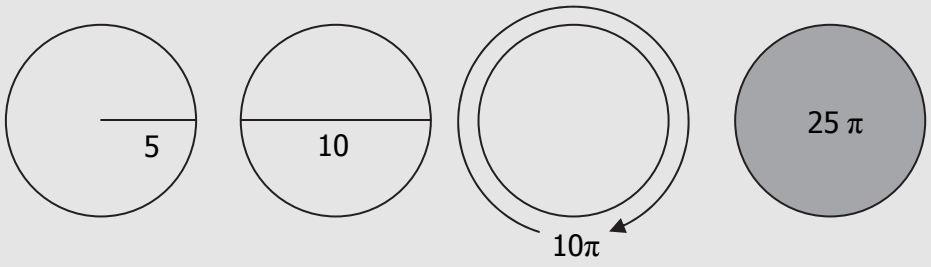
  The value of π is approximately 3.1416.

- **Circumference** is the distance around the edge of the circle.

  Circumference $= 2 \times \pi \times r = \pi \times d$

- The **area** of a circle is found using the formula:

  Area $= \pi \times r^2$



The radius of the circle above is 5.

The diameter of the circle is 10. $2 \times r \Rightarrow 2 \times 5 \Rightarrow 10$

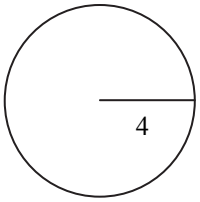
The circumference of the circle is 10π. $2 \times \pi \times r \Rightarrow 2 \times \pi \times 5 \Rightarrow 10\pi$

The area of the circle is 25π. $\pi \times r^2 \Rightarrow \pi \times 5^2 \Rightarrow 25\pi$

TRY IT OUT

Find the circumference of the circles below. Leave in terms of π.

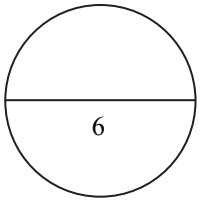
1.



Radius = 4

Circumference =

2.

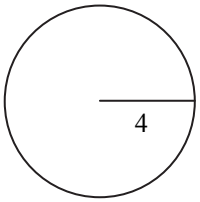


Diameter = 6

Circumference =

Find the area of each of the circles below:

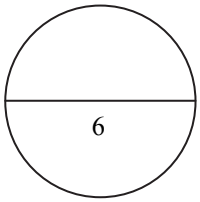
3.



Radius = 4

Area =

4.

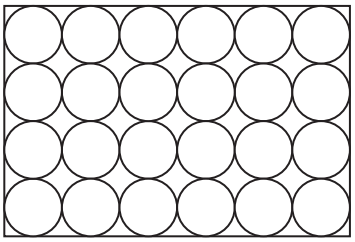


Diameter = 6

Area =

PUT IT TOGETHER QUESTION 1: MIDDLE LEVEL QUESTIONS 2-3: UPPER LEVEL

1. 24 coins are placed on the base of a rectangular display case, as illustrated below. The radius of each coin is half an inch.



Which of the following is the area, in square inches, of the base of the display case?

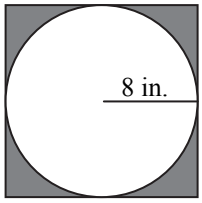
(A) 6
(B) 12
(C) 24
(D) 48

---

2. Circle A has a radius of 6. If Circle B has a radius half the length of the radius of Circle A, then the circumference of Circle B is

(A) 3π
(B) 6π
(C) 9π
(D) 12π

3. A circle with radius 8 inches is inscribed in a square, as shown below.



What is the area, in square inches, of the shaded region?

(A) $64 - 16\pi$
(B) $64 - 64\pi$
(C) $256 - 16\pi$
(D) $256 - 64\pi$

Simplifying and Manipulating

- Frequently, the key to quantitative comparisons is to simplify or manipulate the quantities or the information given. The simplified quantities will make the problem easier.

  Remember, if both quantities are the same, even with a variable, the answer is (C).

Column A		Column B
	$u + w = -2$	
$5u + 5w$		-11

In order to compare the two quantities here, you need to find the value of $5u + 5w$.

You can rewrite $5u + 5w$ as $5(u + w)$.

You know from the information given that $u + w = -2$. Therefore, $5(u + w)$ is the same as $5(-2)$. So, Column A is equal to 5×-2, or –10, which is greater than Column B, which is –11.

(A) is the correct answer.

TRY IT OUT

	Column A	Column B	
1.	0	$0 \div 10$	A B C D
2.	10	$\frac{1}{10} \times \frac{100}{10}$	A B C D
3.	$2 \times (3-1)$	$(2 \times 3) - 1$	A B C D
4.	2	$\frac{1}{2} + \frac{3}{2}$	A B C D
5.	$x - w$	0	A B C D

$x = 2$
$w = 3$

6.	$\frac{5}{7} + .01 + \frac{1}{4} + x$	$\frac{1}{4} + .01 + \frac{6}{7} + \text{x}$	A B C D
7.	$\frac{1}{2} \times \frac{4}{3}$	$\frac{2}{3}$	A B C D
8.	10% of x	5% of $2x$	A B C D
9.	$\sqrt{3} + \sqrt{3}$	$\sqrt{3} \times \sqrt{3}$	A B C D

Positive or Negative

- Even if you can't define a word, you might have a sense of whether it has a positive or negative meaning.

  Determine whether each answer choice is positive or negative and eliminate the ones that don't match the stem word. Put a "+" or a "–" next to the words to keep track. You can then guess from the remaining choices.

  Keep in mind that some words are not necessarily positive or negative.

  – DESPICABLE:

  – (A) disdainful
  \+ (B) admirable
  \+ (C) responsible
  ± (D) animated

  If you know that *DESPICABLE* is a negative word, you know that another negative word must be the answer. You can eliminate (B) and (C) because they are positive words. You can eliminate (D) because it is neither positive nor negative. You can then assume that (A) must be the answer.

- Remember, checking whether words are positive or negative should NOT be your first strategy for synonyms questions. This is a backup strategy, and it should be used when you cannot use other strategies because you don't know the meanings of the words.

TRY IT OUT

Put a + (for positive) or a – (for negative) next to each of the following words:

1. crass
2. caustic
3. beneficial
4. malice
5. squalor
6. paltry
7. vivacious
8. harmonious
9. ghastly
10. bountiful

Put a + or a – by the stem word and then make your best guess on the following synonyms:

11. BENEFICENT:
    (A) angry
    (B) sneaky
    (C) generous
    (D) attractive

12. NOXIOUS:
    (A) magnificent
    (B) religious
    (C) healthful
    (D) harmful

13. BANAL:
    (A) prolific
    (B) generous
    (C) soothing
    (D) bland

Anticipating the Answer

- ❑ After you read the sentence, but before you look at the answer choices, try to fill in the blank with a word of your own. After you've done this, pick the answer choice that most closely matches your anticipated answer.

> After his injury, Jeremy was so ------- that he was unable to play during the most important game of the season.
>
> (A) helpless
> (B) wounded
> (C) irritated
> (D) raw
>
> Before looking at the answers, try to fill in the blank with your own word. In this sentence, the blank is likely a word that means "injured." (B) is the best match, and it's the correct answer.

- ❑ Find the clues.

The most important skill in solving sentence completions is the ability to identify the words and phrases in the sentence that tell you what you're looking for. Every sentence will have clues in it.

> You will *never* see: Amy's father is ------- .
> (A) sick
> (B) poor
> (C) healthy
> (D) wealthy
>
> In this case, there is no way to tell what the correct answer is, because there are no clues to the blank.
>
> Instead, you'll see:
>
> Amy's father is ------- and, therefore, will need medication.
>
> (A) sick
> (B) poor
> (C) healthy
> (D) wealthy
>
> "Will need medication" is the clue, which should guide you to answer (A).

TRY IT OUT

Underline the clues in the sentence, and fill in the blank(s) with words of your own:

1. Having skipped both breakfast and lunch, by dinner Dean was _____ .

2. Because he is so _____, we can never tell what he will do next.

3. When performing a science experiment that involves dangerous chemicals and gases, it is important to be very _____ each step of the way; otherwise, you may end up injuring yourself.

4. Unlike his brother, Sam, who became very _____during the long car trip, Martin maintained his sense of humor and keen interest in everything they passed.

5. Gerard considered himself a highly _____ baseball player and took it upon himself to instruct all his friends on how to catch, throw, and hit a baseball.

6. Michael's _____ made him an ideal candidate to become a peer counselor, while Norman's _____ eliminated him from consideration for the position.

Anticipating the Answer

- Before looking at the answer choices, try to think of the answer in your head. Try not to look at the answer choices until you know what the answer should be. Then find the answer that most closely matches your anticipated one.

  Anticipating the answer will save you time on the Reading Comprehension section because it allows you to find the answer by solving the question once.

- On many ISEE reading questions, there are multiple answer choices that **could be** correct, but there is only one **best** answer. If you try to test if you can prove the answer choices correct, you might get stuck with several answers that seem right. Instead, focus on finding your own best answer first.

Surprisingly, the history of flea circuses begins with watchmaking. In a display of their incredibly precise metal-working skills, watchmakers created tiny props for fleas. This led to the first flea circuses. Fleas were used as the performers because of their strength and availability. At the time, before effective pest control, fleas were a common part of everyday life.

According to the passage, flea circuses were created in order to

(A) pass the time while watchmakers were not busy working.
(B) test the capabilities of different metals and metal-working tools.
(C) demonstrate the talents of watchmakers.
(D) exhibit the incredible skill of fleas.

This might be a challenging question, because several answer choices could be correct.

(A) could be right, because making flea circuses could have been a hobby to pass the time.

(B) could be right, because the watchmakers could test new materials and methods by creating the tiny props.

(D) could be right, because the circuses would show the strength of fleas.

If you ignore the answer choices and just consider the question, it is clear from the information in the passage that flea circuses were made to display watchmakers' skills. (C) is the answer that best matches this idea.

TRY IT OUT

Many organisms have developed incredible adaptations for the environments in which they live. The most impressive examples are classified as "extremophiles," which are organisms that can thrive in conditions that are too harsh for most forms of life. One of the most well-known and unique extremophiles is the tardigrade, also known as the "water bear" or "moss piglet." This microscopic animal looks like a cross between a grub and a gummy bear. Tardigrades can survive in environments that would be lethal to any other animal. They can live in termperatures colder than -400 degrees and hotter than 300 degrees Fahrenheit. They can also survive without water for nearly a decade. This is necessary because tardigrades commonly live in puddles and moss, which often dry out. They can reduce their metabolism to less than one-thousandth of their normal rate, and will return from this dormant state when they have a supply of water. Tardigrades have been found in many of the harshest environments on earth, such as boiling hotsprings and arctic ice. They can even survive the vacuum of space! These amazing creatures show the surprising resilience of life. Research on extremophiles has led to new discoveries that allow scientists to work in conditions that would be too severe for our own bodies.

1. The author mentions "boiling hotsprings" (line 10) as an example of

   _____

   _____

   _____

2. In line 8, "state" most nearly means

   _____

   _____

   _____

3. It can be inferred from the passage that the tardigrade can survive dehydration by

   _____

   _____

   _____

4. This passage is primarily about

   _____

   _____

   _____

PUT IT TOGETHER

Many organisms have developed incredible adaptations for the environments in which they live. The most impressive examples are classified as "extremophiles," which are organisms that can thrive in conditions that are too harsh for most forms of life. One of the most well-known and unique extremophiles is the tardigrade, also known as the "water bear" or "moss piglet." This microscopic animal looks like a cross between a grub and a gummy bear. Tardigrades can survive in environments that would be lethal to any other animal. They can live in temperatures colder than -400 degrees and hotter than 300 degrees Fahrenheit. They can also survive without water for nearly a decade. This is necessary because tardigrades commonly live in puddles and moss, which often dry out. They can reduce their metabolism to less than one-thousandth of their normal rate, and will return from this dormant state when they have a supply of water. Tardigrades have been found in many of the harshest environments on earth, such as boiling hotsprings and arctic ice. They can even survive the vacuum of space! These amazing creatures show the surprising resilience of life. Research on extremophiles has led to new discoveries that allow scientists to work in conditions that would be too severe for our own bodies.

1. The author mentions "boiling hotsprings" (line 10) as an example of

   (A) new scientific discoveries.
   (B) a source of water for dehydrated tardigrades.
   (C) how organisms affect their environment.
   (D) an extreme environment in which most organisms cannot survive.

2. In line 8, "state" most nearly means

   (A) area.
   (B) declare.
   (C) condition.
   (D) public.
   (E) structure.

3. It can be inferred from the passage that the tardigrade can survive dehydration by

   (A) shrinking its size.
   (B) slowing its bodily processes.
   (C) living on moss.
   (D) melting ice.

4. This passage is primarily about

   (A) the hardiness and adaptability of organisms.
   (B) why tardigrades do not need water.
   (C) the world's smallest living animal.
   (D) the benefits of biological research.

The Introduction

- The purpose of the introduction is to state the main idea of your essay. In the introduction, you define and limit what you are going to discuss.

  The introduction is like a preview of your whole essay. It should mention the example you'll be discussing and it should state your thesis.

- **Basic Introduction:** The simplest way to begin your essay is to state your thesis, which is usually an answer or response to the essay prompt. Continue your introduction by describing your example and how it connects to your thesis. You don't need many details or specifics yet; these will be covered in the essay body.

  > A hobby I have that makes me special is rock climbing. This sport has taught me a lot about myself, because it has made me aware of my physical and mental strengths. It has also made me stronger and braver, and it has been an inspiration for how I live my life.

- **Inverted Funnel:** A more advanced method for writing an introduction is to begin with a general statement about the topic, then narrow the topic until you end with a statement of your thesis, specifically identifying the examples.

  > I grew up playing basketball, swimming, and attempting to play softball. All of these sports gave me an appreciation for healthy competition and team camaraderie, but I never felt a special connection with any of these. Rock climbing is different. It's individualistic, which means you have to trust and rely on your own strength and intelligence. Because of this, it teaches you a lot about your strengths and your limits. Climbing has taught me more about myself than any team sport I've played and it continues to do so.

TRY IT OUT

Write an introduction for an essay using each of the following prompts:

1. *Describe an experience that has inspired you.*

2. *Discuss a problem in the world that you would like to solve and how you would solve it.*