Graphing Polynomials

First, make sure the polynomial is written as factors, if possible.
For simplicity, I suggest determining the elements of the polynomial in the order below.

DOMAIN:

- *Always* "All Real Numbers;" $(-\infty, \infty)$; $-\infty < x < \infty$
  - *unless* there is a variable:
    - in the denominator
      - set the denominator $\neq 0$
      - solve for $x \neq$
    - inside of an even root
      - set the expression inside the root ≥ 0
      - solve for $x \geq$
    - inside of a logarithm
      - set the expression inside the log > 0
      - Solve for x >

VERTICAL ASYMPTOTE (VA):

1. *Ignore* the factors that *cancel*
2. Set the remaining factors in the denominator equal to zero
   - If no variable in denominator, there is no VA
3. Solve for x = (this is the VA)

HOLE:

1. Set factors that cancel equal to zero
   - If no factors cancel, there are no holes
2. Solve for x = (this is the hole)
3. Plug this x-value into the simplified equation to solve for y

HORIZONTAL ASYMPTOTE (HA):

Write the numerator and denominator in standard form (decreasing exponential order)

$\frac{ax^n}{bx^m}$ (use the term with the greatest exponent in the numerator)
(use the term with the greatest exponent in the denominator)

if n = m, HA is $y = a / b$
if n > m, no HA
if n < m, HA is $y = 0$

SLANT ASYMPTOTE:

Write the numerator and denominator in standard form (decreasing exponential order)

ax^n (use the term with the greatest exponent in the numerator)
bx^m (use the term with the greatest exponent in the denominator)

if $n = m + 1$, there is a Slant Asymptote, which can be found:

1. Do long division with the polynomials in the numerator as the dividend and denominator as the divisor
2. *Ignore* the *remainder*, and the polynomial part of the quotient is the equation of the slant asymptote

*NOTE: A graph can have both a vertical and a slant asymptote, but it cannot have both a horizontal and a slant asymptote

RANGE:

- *Always* "All Real Numbers," $(-\infty, \infty)$; $-\infty < y < \infty$
    - *unless* there is a HA
        - then "except $y \neq$ ___"
    - *unless* there is a limit on the y-values
        - can be determined by limits of the Domain
        - can be determined by graphing

X-INTERCEPT:

1. *Ignore* factors that *cancel*
2. Set remaining factors in numerator equal to zero
3. Solve for $x =$ (this is the *x*-intercept)

Y-INTERCEPT:

1. Plug in *zero* for all *x*'s
2. Solve for $y =$ (this is the *y*-intercept)

Once you graphed all of this information, you can calculate and plot several other points on the graph in order to determine the shape of the graph. Aim to have at least 2 points in each section of the graph that is marked off by the vertical asymptote(s).