Asymptotes and Holes

Vertical Asymptote (VA):

1. Write the polynomial as factors, if possible.
2. *Ignore* the factors that *cancel*.
3. Set the *remaining* factors in the *denominator* equal to zero. If there is no variable in the denominator, then there is no VA.
4. Solve for x = (this is the VA).

Hole:

1. Write the polynomial as factors, if possible.
2. Set the factors that *cancel* equal to zero. If no factors cancel, then there are no Holes.
3. Solve for x.
4. Plug this x-value into the simplified equation to solve for y. This gives the x,y coordinates of the Hole.

NOTE: The domain of a function is all real numbers except the x-values of the VA(s) and the Hole(s).

Horizontal Asymptote (HA):

Write the numerator and denominator in standard form (decreasing exponential order).

$$\frac{ax^n}{bx^m}$$

If n = m, HA is $y = \frac{a}{b}$

If n > m, no HA

If n < m, HA is $y = 0$

NOTE: Questions that ask what value a function approaches as x approaches positive or negative infinity are technically asking for the y-value of the HA.

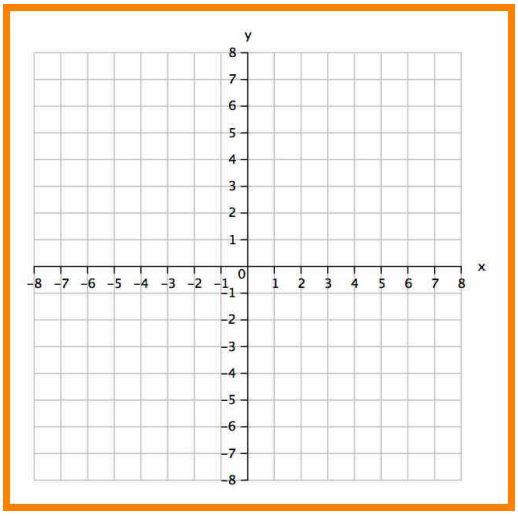
Slant Asymptote (SA):

Write the numerator and denominator in standard form. If the degree (highest exponent) of the numerator is *exactly one more* than the degree of the denominator, then there is a Slant Asymptote (and therefore no HA).

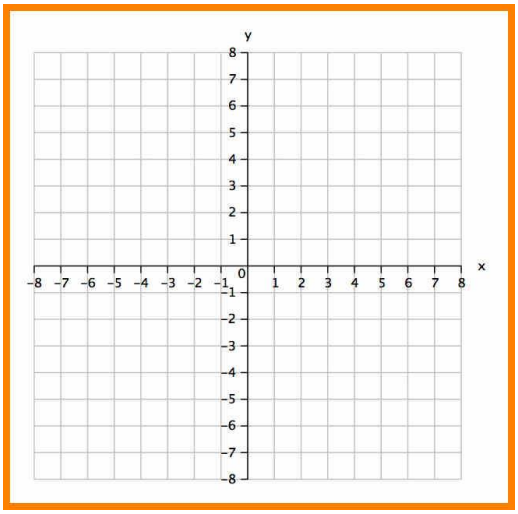
1. Do long division with the polynomial in the numerator as the dividend and the denominator as the divisor.
2. *Ignore* the *remainder*. The polynomial part of the quotient is the equation of the Slant Asymptote (write it as y =).

Practice:

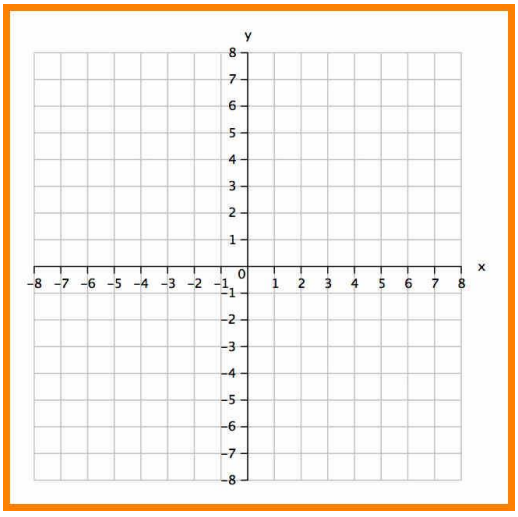
1. f(x) = $\frac{1}{3x^2 + 3x - 18}$



2. f(x) = $\frac{x^2 + x - 6}{-4x^2 - 16x - 12}$



3. f(x) = $\frac{x^3 - 9x}{3x^2 - 6x - 9}$



Answer Key:

1.

VA: $x = -3$ and $x = 2$

Hole: None

HA: $y = 0$

SA: None

2.

VA: $x = -1$

Hole: (-3, -5/8)

HA: $y = -1/4$

SA: None

3.

VA: $x = -1$

Hole: (3, 3/2)

HA: None

SA: $y = 1/3x + 2/3$