

Verifying Trigonometric Identities

An “identity” is an equation or statement that is always true. Verifying an identity is different from solving an equation. You should not plug in values for the angles because there are an infinite number of coterminal angles that can make the equation true. You should not move terms from one side to the other because you are verifying not solving. No single method works for verifying all identities, but following certain steps will help.

Recommended Steps for Verifying a Trigonometric Identity:

- 1) Work on the side that is more complicated. Leave the other side alone.
- 2) Use algebra to simplify if possible (factor, get common denominator, multiply numerator and denominator by conjugate, etc).
- 3) Convert all expressions to sines and cosines using Trigonometric Identities.
- 4) Use algebra to simplify as much as possible.
- 5) Use relevant Trigonometric Identities to simplify as much as possible.
- 6) Repeat Steps 4 & 5, alternating between algebra and identities, as much as is needed until the side you are working on is equivalent to the side you left alone.

Most Common Trigonometric Identities Used for Verifying:

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Examples:

$$1) \tan \theta \sin \theta = \sec \theta - \cos \theta$$

$$2) \frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$$

$$3) \tan \theta + \cot \theta = \sec \theta \csc \theta$$

$$4) \frac{\csc^2 \theta + 2 \csc \theta - 3}{\csc^2 \theta - 1} = \frac{\csc \theta + 3}{\csc \theta + 1}$$

$$5) (\sec \theta - \tan \theta)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$$