

FINANCIAL CALCULATIONS

I : Interest (\$)
 A_0 : Initial Amount/Deposit (\$)
 r : Interest Rate (% , but converted to a decimal)
 t : Time (years)
 k : Number of Compounding Periods or Monthly Deposits Per Year
 $i = \frac{r}{k}$: Periodic Interest Rate (% , but converted to decimal)
 A_f : Future Value (\$)
 A_p : Present Value (\$)
 P : Periodic Payment (\$)

Simple Interest:

The simple interest I earned on an initial deposit A_0 in an account paying an annual interest rate r for a length of time t is given by the formula:

$$I = A_0 * r * t$$

Example: Find the simple interest earned on a deposit of \$5,750 that earns an annual interest rate of 4.5% after 3.5 years.

$$\begin{aligned}
 I &= A_0 * r * t \\
 I &= (5750)(0.045)(3.5) \\
 I &= \$905.63
 \end{aligned}$$

Compound Interest:

An account earns compound interest when the interest also earns interest.

An initial amount A_0 earning interest compounded k times a year for t years at rate r , will grow to the future value A_f according to the formula:

$$A_f = A_0 (1 + i)^{kt} = A_0 \left(1 + \frac{r}{k}\right)^{kt}$$

- “Annually”: $k = 1$
- “Monthly”: $k = 12$
- “Semiannually”: $k = 2$
- “Weekly”: $k = 52$
- “Quarterly”: $k = 4$
- “Daily”: $k = 360$ or 365

Example: If \$10,000 is invested in an account paying 8%, compounded quarterly, what will be its future value after 17 years?

$$\begin{aligned}
 A_f &= A_0 \left(1 + \frac{r}{k}\right)^{kt} \\
 A_f &= 10000 \left(1 + \frac{0.08}{4}\right)^{4 \cdot 17} \\
 A_f &= \$38,442.51
 \end{aligned}$$

Annuity:

Annuities are financial plans that involve a series of payments. For instance, monthly mortgage payments and regular contributions to retirement plan are both parts of annuities.

The present value A_p of an ordinary annuity with payments of P dollars withdrawn regularly k times each year for t years at the end of each period, with interest compounded k times per year at an annual interest rate r is:

$$A_p = P * \frac{1 - (1 + \frac{r}{k})^{-kt}}{\frac{r}{k}} = P * \frac{1 - (1 + \frac{r}{k})^{-kt}}{\frac{r}{k}}$$

Example: What must you invest today at 6% interest compounded annually so that you can withdraw \$5,000 at the end of each year for the next 5 years?

$$A_p = P * \frac{1 - (1 + \frac{r}{k})^{-kt}}{\frac{r}{k}}$$

$$A_p = 5000 * \frac{1 - (1 + \frac{.06}{1})^{-1*5}}{\frac{.06}{1}}$$

$$A_p = \$21,061.82$$

The future value A_f of an ordinary annuity with deposits of P dollars made regularly k times each year for t years at the end of each period, with interest compounded k times per year at an annual interest rate r is:

$$A_f = P * \frac{(1 + \frac{r}{k})^{kt} - 1}{\frac{r}{k}} = P * \frac{(1 + \frac{r}{k})^{kt} - 1}{\frac{r}{k}}$$

Example: Assume that a savings account pays 12% annual interest, compounded monthly. Also assume that each month for the next 5 years, \$100 will be deposited in that account. Determine the future value of the annuity.

$$A_f = P * \frac{(1 + \frac{r}{k})^{kt} - 1}{\frac{r}{k}}$$

$$A_f = 100 * \frac{(1 + \frac{.12}{12})^{12*5} - 1}{\frac{.12}{12}}$$

$$A_f = \$8,166.97$$

Amortization:

An amortization involves the repayment of a bank loan spread over several equal payments.

To calculate the periodic payments P required to repay an amount A_0 is given by:

$$P = \frac{A_0 * i}{1 - (1 + i)^{-kt}} = \frac{A_0 * \frac{r}{k}}{1 - (1 + \frac{r}{k})^{-kt}} .$$

Example: What is the monthly payment for a 15-year mortgage of \$200,000 compounded monthly at 10.8%?

$$P = \frac{A_0 * \frac{r}{k}}{1 - (1 + \frac{r}{k})^{-kt}}$$

$$P = \frac{(200000)(\frac{0.108}{12})}{1 - (1 + \frac{0.108}{12})^{-12*15}}$$

$$P = \$2,248.14$$