

## QUADRATIC WORD PROBLEMS

General quadratic equation:

$$y = ax^2 + bx + c$$

General quadratic equation of projectile motion:

$$h(t) = \frac{1}{2}gt^2 + v_0t + h_0$$

$h(t)$ : height as a function of time (m or ft)

$g$ : approximate acceleration of gravity is  $-9.81 \text{ m/s}^2$  or  $-32.17 \text{ ft/s}^2$

$t$ : time (s)

$v_0$ : initial velocity (m/s or ft/s)

$h_0$ : initial height (m or ft)

**Question 1: “What time does the object reach its maximum height?”**

- Use Vertex Formula:  $t = \frac{-b}{2a}$
- $t$  is the time in seconds (s)

**Question 2: “What is the object’s maximum height?”**

- From Question 1, you determined the time the object reaches its maximum height. Now substitute this time for  $t$  into the original equation.
- Solve for  $h(t)$

**Question 3: “At what times does the object reach a height of \_\_\_?”**

- Substitute given height into  $h(t)$  of the original equation.
- Rearrange the equation so that it equals zero.
- Solve for  $t$  using factoring or the Quadratic Formula.

NOTE: Because the projectile graph is a parabola shape, there are two times when the parabola reaches that height (once on the way up and again on the way down)

**Question 4: “What height is the object at \_\_\_ seconds after it is launched?”**

- Substitute given time for  $t$  into the original equation.
- Solve for  $h(t)$ .

**Question 5: “At what time does the object reach the ground?”**

- Substitute zero for  $h(t)$  into the original equation (at ground level, height = 0m).
- Because the parabola opens downward, the  $t$ -intercepts are when  $h$  equals zero.
- To determine the  $t$ -intercepts, plug in the  $a$ ,  $b$ , and  $c$  values from the original equation into the Quadratic Formula and solve for the two  $t$ -values.

NOTE: The smaller  $t$ -value is when the object leaves the ground (this may be negative if the object started at a height greater than 0m or 0ft; if so, ignore this value), and the larger  $t$ -value is when the object lands on the ground.

**Question 6: “What is the average speed/velocity of the object \_\_\_ seconds after it is launched?”**

NOTE: The slope of a distance graph is the velocity (m/s or ft/s)

- Substitute the initial time (0s) for  $t$  into the original equation and solve for  $h_0$  (this is also the  $c$ -value).
- Substitute the given time for  $t$  into the original equation and solve for  $h(t)$ .
- Calculate the slope using the slope formula:  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{h(t) - h_0}{t - 0}$
- The slope is the velocity in m/s or ft/s

**Question 7: “What is the average speed/velocity of the object between \_\_\_ seconds and \_\_\_ seconds?”**

- Substitute the first given time ( $t_1$ ) for  $t$  into the original equation and solve for  $h(t)$  (which =  $h_1$ )
- Substitute the second given time ( $t_2$ ) for  $t$  into the original equation and solve for  $h(t)$  (which =  $h_2$ )
- Calculate the slope using the slope formula:  $m = \frac{h_2 - h_1}{t_2 - t_1}$

**Question 8: “What is the speed/velocity of the object when/right before it hits the ground?”**

- From Question 5, you determined the time that the object hits the ground; call this time  $t_2$  and this height  $h_2$  (NOTE:  $h_2 = 0$ )
- Now subtract 0.001 s from that time ( $t_2$ ) to get  $t_1$ , which is the time right before the object hits the ground.
- Substitute  $t_1$  into the original equation and solve for  $h(t)$  (which =  $h_1$ )
- Calculate the slope using the slope formula:  $m = \frac{h_2 - h_1}{t_2 - t_1}$

**Question 9: “What is the instantaneous speed/velocity of the object at \_\_\_ seconds?”**

- Substitute the given time ( $t_1$ ) for  $t$  into the original equation and solve for  $h(t)$  (which =  $h_1$ )
- Now subtract 0.001 s from that time ( $t_1$ ) to get  $t_2$ , which is the time right before that instant
- Substitute  $t_2$  into the original equation and solve for  $h(t)$  (which =  $h_2$ )
- Calculate the slope using the slope formula:  $m = \frac{h_2 - h_1}{t_2 - t_1}$