FACTORING POLYNOMIALS

1) ARE THERE COMMON FACTORS?

Factor out the **Greatest Common Factor (GCF)** of the expression, if one exists. This will make it simpler to factor the remaining expression. Be careful NOT to drop this GCF, as it is still part of the expression's answer.

Examples:
$2x^2 – 6xy^2 = 2xy(x - 3y)$
$15x^2y + 12xy^2 + 3y = 3y(5x^2 + 4x + 1)$

2) DOES THE EXPRESSION HAVE ONLY 2 TERMS?

a) Is the expression a **difference of perfect squares**? If so, the factors are the sum and the difference of the square roots of the terms.

Examples:
$x^2 – 25 = (x + 5)(x - 5)$
$16x^2 – 25y^2 = (4x + 5y)(4x - 5y)$
$2x^4 – 32 = 2(x^4 – 16) = 2(x^2 + 4)(x^2 – 4) = 2(x^2 + 4)(x + 2)(x – 2)$

b) If the expression is not a difference of perfect squares, just factor out the GCF like in #1 above

Examples:
$25x^2 + 5x = 5x(5x + 1)$
$2x^3 – 32x = 2x(x^2 – 16) = 2x(x – 4)(x + 4)$

3) DOES THE EXPRESSION HAVE EXACTLY 3 TERMS?

$f(x) = y = \mathbf{a}x^2 + \mathbf{b}x + \mathbf{c}$

a = "leading coefficient"
b = "coefficient of the second term"
c = "last term" or "constant"

a) Is the leading coefficient (a) equal to 1? If so, you are looking for two numbers that multiply to the last term (c) and add to the coefficient of the middle term (b).

Examples:

$x^2 - 6x + 8 = (x - 2)(x - 4)$

$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$

b) Is the leading coefficient (a) equal to 1, but there are not two numbers possible that multiply to the last term (c) and add to the coefficient of the middle term (b)? Then you can try "**Completing the Square**." Follow these steps:

1st: Group the first two terms together by putting parentheses around them
2nd: Add a plus blank in the parentheses after the 2nd term, then add a minus blank after the last term (c)
3rd: Divide the coefficient of the 2nd term (b) by 2, and then square that quotient; write this number in the blanks
4th: Factor the expression in the parentheses using the technique in #3a above (HINT: the quotient of b/2 is in the factor for each)
5th: Combine the like terms outside of the parentheses
6th: Combine the two like factors together

Example:

$x^2 + 2x + 2$

$(x^2 + 2x + \_\_\_) + 2 - \_\_\_\_$ $(2/2 = 1;\ 1^2 = 1)$

$(x^2 + 2x + \underline{1}) + 2 - \underline{1}$

$(x + 1)(x + 1) + 1$

$(x + 1)^2 + 1$ ← This is known as "**Vertex Form**" $(x – h)^2 + k$, and thus the vertex in this case is (-1, 1)

c) Is the leading coefficient (a) equal to -1? If so, you need to factor out the negative sign first. Then look for two numbers that multiply to the last term (c) and add to the coefficient of the middle term (b).

Examples:

$- x^2 + 8x + 20 = - (x^2 - 8x - 20) = - (x + 2)(x - 10)$

$- x^2 + 6x - 9 = - (x^2 - 6x + 9) = - (x - 3)(x - 3) = - (x - 3)^2$

d) Is the leading coefficient (a) a common factor? If so, you need to factor out the GCF first. Then look for two numbers that multiply to the last term (c) and add to the coefficient of the middle term (b).

Example:

$-3x^2 + 30x - 75x = -3 (x^2 - 10x + 25) = -3 (x - 5)(x - 5) = -3 (x - 5)^2$

e) If the leading coefficient (a) is not equal to 1, -1, or a GCF, then you need to follow these steps:

1st: Calculate the product of the leading coefficient (a) and the last term (c)
2nd: Look for 2 numbers that multiply to that product (a*c) and add to the coefficient of the middle term (b)
3rd: Divide each of those 2 numbers by the leading coefficient (a) and simplify the fraction if possible.
4th: If the fraction does not simplify, swing the denominator in the front of the *x* for that factor.

Example 1:
$2x^2 + x - 6$
$(2)(-6) = -12$ -> so the 2 numbers are 4 & -3 -> 4/2 = 2, but -3/**2** does not simplify, so…
$(x + 2)(2x - 3)$

Example 2:
$4x^2 - 19x + 12$
$(4)(12) = 48$ -> so the 2 numbers are 16 & 3 -> 16/4 = 4, but 3/**4** does not simplify so…
$(x + 4)(4x + 3)$

NOTE: If there are not two numbers that multiply to the product of the leading coefficient and the last term (a*c) as well as add to the coefficient of the middle term (b), then you will need to use the "**Quadratic Formula**":

$$\mathbf{x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}}$$

Example:

$9n^2 + 4n - 16$

$$x = \frac{-4 \pm \sqrt{(4^2 - 4(9)(-16))}}{2(9)} =$$

$$x = \frac{-4 \pm \sqrt{(592)}}{18} =$$

$$x = \frac{-4 \pm 4\sqrt{37}}{18} =$$

$$x = \frac{-2 \pm 2\sqrt{37}}{9}$$

4) DOES THE EXPRESSION HAVE EXACTLY 4 TERMS?

You can try “**Factoring by Grouping**.” Follow these steps:

1st: Group the first two terms together and the last two terms together by putting parentheses around the pairs

2nd: Factor out the GCF from the first pair and then factor out the GCF from the second pair

3rd: The inside of each parenthesis should be the same, so factor that out and put the remaining terms in another parenthesis

Example 1:

$12x^3 – 21x^2 + 28x – 49 =$

$(12x^3 – 21x^{2)} + (28x – 49) =$

$3x^2(4x – 7) + 7(4x – 7) =$

$(4x – 7) (3x^2 + 7)$

Example 2:

$8x^3 – 64x^2 + x – 8 =$

$(8x^3 – 64x^2) + (x – 8) =$

$8x^2(x – 8) + \mathbf{1}(x – 8) =$

$(x – 8) (8x^2 + 1)$

Example 3:

$105x^3 + 175x^2 – 75x – 125 =$

$(105x^3 + 175x^2) – (75x \mathbf{+} 125) =$

$35x^2(3x + 5) – 25(3x + 5) =$

$(3x + 5) (35x^2 – 25)$